Anything Goes in Squid Game

Sequential Voting with Informed and Uninformed Voters[*]

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Abstract

We consider a model of common-value sequential voting where voters are differentiated in their information. The intuition from the simultaneous-voting case—uninformed voters vote so as not to influence the outcome—suggests that the pivotal voter, unless being informed, would not like to end the voting. This would imply that voting takes a long time if voters are unlikely to be informed. However, we find that any voting outcome, including short voting, can arise in an equilibrium that uses consistent strategies, i.e., the informed voters follow their information. Despite such variety of equilibria, all of them induce the same expected payoff to the voters. There also exist inconsistent-strategy equilibria that yield higher expected payoffs than under any consistent strategy profile. One of such equilibria yields the highest feasible expected payoff.

Keywords: sequential voting, common value, information aggregation, delegation

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1   Introduction

In episode 2 of the blockbuster Netflix show *Squid Game*, voting was held to determine if the 201 players in the game would continue or stop the game. The votes were cast sequentially, in the order of the players' jersey numbers. All players watched each other's vote, and the current vote counts were displayed on a screen in front of them. The voting was close and took a long time, and the 201st voter, a fragile elderly man, took his turn.[1]

Besides the *Squid Game* show, sequential voting is used on various real-life occasions. Most importantly, it is a common method employed in political decision-making worldwide. For example, in the Japanese Diet, the roll call vote is done by each Diet member coming up to the front stage and giving to a staff called *Sanji* a wooden plate that is either white (representing the "yes" vote) or blue (representing the "no" vote). The *Sanji* would then put the plates on a meter according to their color, and this meter is visible to everyone in the Diet. The votes can easily be counted at any moment because there are lines on the meter per every five plates. A similar procedure of roll call vote is used in the United Nations as well as in the US Senate, among others. In those settings, voters would not be perfectly informed of the favorability of options, where the degree of uncertainty may differ across voters. For example, most Senate or Diet members would face uncertainty about the future evolution of the economy, while there may sometimes exist those who possess inside information. In *Squid Game*, it is still episode 2 of the show when the vote was held, so the players were unsure about the environment as well as what other players would know. How would voters vote if they are uncertain about the favorability of the options?

Voter behavior with uncertainty has been discussed in static frameworks, notably by the seminal work by Feddersen and Pesendorfer (1996). They analyzed a common-value framework with informed and uninformed voters and showed that, in equilibrium, an uninformed voter (that is, she is uncertain which option is better) would prefer to abstain, thereby essentially delegating the decision to other voters who might be informed. If the same incentive as in Feddersen and Pesendorfer (1996) was at work in the sequential setting with no abstention option (as in all the above examples), then we would expect a *long voting*: the pivotal voter, if she was uninformed, would vote against the current majority, thereby having the voting continued

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[1]Which option did he vote? Watch *Squid Game.*

and delegating the decision to the subsequent (possibly informed) voters.

We show that this intuition is not entirely correct. To understand the incentives in the sequential setting, we analyze a stylized model of sequential voting, abstracting away from various aspects that may exist in different contexts of real-world voting. In our model, there are two states of the world, α and β, which realize with equal probability. Voters are either informed or uninformed: conditional on each state, each voter can (independently) be perfectly informed about the state with a small probability, or else they are completely uninformed.[2] Each voter votes for either A or B until a majority has cast votes for one of the options, which determines the outcome. Voters receive payoff 1 if the outcome "matches" the state and 0 otherwise. Our first main result says "anything goes": For any sequence of votes that can feasibly realize in the game, there is an equilibrium that achieves such a sequence if no one is informed. That is, not only long voting but also any length of voting can be an outcome of our voting game.

Why does anything go?[3] In our voting model, Feddersen and Pesendorfer (1996)'s intuition is indeed at work: If a pivotal voter is uninformed, she would like to delegate the decision to the subsequent voters. We call this effect the *delegation effect*. This, however, is only one side of the story. There is another side: *A voter's optimal vote depends on what she has learned from the preceding voters' choices and the effect of her vote on the subsequent voters' choices.* This learning, which arises due to the sequential nature of our model, makes it appealing for the pivotal voter to end the game (because that means she votes for the current majority), and we call this effect the *posterior effect.* If this pivotal voter is supposed to vote to end the voting conditional on being uninformed, the posterior effect implies that it is costly for her not to end the voting. This is because not doing so means that she essentially informs the subsequent voters that she knows the state (even though she does not!), and it is the state that she thinks is less likely. We obtain the "anything goes" result due to the balance of these two effects.

To show the "anything goes" result, we construct equilibria that use "consistent

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[2]This mapping from states to signals follows that of Feddersen and Pesendorfer (1996).

[3]One common reaction to this question is that uninformed voters would assign equal probability to each state so are indifferent, and this must be why anything can happen. It will be clear in the following discussion that this argument is invalid: In fact, these voters do not necessarily assign equal probabilities to the states given their observation. In Section 3.1, we consider this issue in detail.

strategies": the voter votes for A if she is informed of state α and votes for B if she is informed of state β. We further show that any two consistent equilibria induce the same (expected) payoff to voters.

Furthermore, we show the existence of inconsistent equilibria. The welfare in inconsistent equilibria can take a wide range: we provide the tight upper and lower bounds of the equilibrium payoffs achievable by inconsistent equilibria. In particular, the upper bound is the maximum payoff achievable by any social choice function. The welfare from consistent equilibria positions in between these bounds.

Given that the "anything goes" result does not give us any prediction about what happens in our voting game, we seek a way to enrich the model to obtain some prediction. Specifically, we find that the following two modifications to the model would imply a "long voting" as a *unique* equilibrium outcome. First, we assume the last voter trembles slightly (as in *Squid Game*). Second, we assume that each voter experiences a small cost by ending the game. We show that each of these two modifications tips over the balance of the posterior effect and delegation effect in favor of the latter, and the voting goes to the last voter under any equilibria if every voter is uninformed.

Our model is stylized to capture the essence of the implications of uncertainty in sequential voting. As such, for each of the real-world voting applications, there are ways in which the model may not perfectly fit. For example, in *Squid Game*, although our model may perfectly capture the timing of moves and observation structure, it could be questionable if our payoff function represents the episode's situation well.[4] In the United Nations, some countries may be known to have different information and preferences than others. In the US Senate and Japanese Diet, different politicians may belong to different parties and hence may have different preferences. These voters, however, may still have common uncertainty about the state of the world, and the

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[4]One (admittedly farfetched) interpretation is the following: In the show, all players were in serious financial debt. If the players had continued the game, they would have received a large monetary prize if they had won (otherwise, they would have been killed). If they had stopped the game, they would not have received money and gone back to a life of financial ruin. No one was supposed to know what sort of games they would play to determine the winner upon continuation. In a sense, every player's situation was similar in terms of the expected payoff from each option. Of course, players might have different beliefs from each other about the likelihood of their own winning, which would have introduced some private-value component to the model. At the same time, since the players did not know each other very well, they could be unsure about what others knew, and this provides the possibility that the players believed that some other players could be more informed.

4

voting may have a common-value component. The fact that our model is stylized enables us to capture the key incentives faced by the voters with uncertainty in a common-value sequential environment by abstracting away from various details.

There are many ways to enrich the model. In Section 6, we provide various discussions and generalizations of the model. For example, we discuss how robust our "anything goes" result is to various types of asymmetries in the model and to the case when voters can only observe vote counts, not the entire history of voting.

1.1 Literature Review

We ask how the differentiated precision of signals across voters affects the behavior in a sequential voting setup. This question is asked in the simultaneous voting setting by Feddersen and Pesendorfer (1996) that has an otherwise similar setup as ours. As we discussed, the fact that the voting is sequential changes the picture quite a bit.

Dekel and Piccione (2000)'s seminal work on sequential voting considers an information structure and payoffs that are more general than ours. Their focus, however, is on whether history-independent strategies that are equilibria under simultaneous voting also constitute equilibria under a sequential setting. They also have an analysis of a common-value case like ours, but they restrict attention to the case of two signals.[5] We, in contrast, consider history-dependent strategies, and analyze the case with three signals (two informative and one uninformative signals) to address the question of how the differentiated precision of signals across voters affects the behavior.[6]

One can think of our model as that of herding. The main difference is that, in the canonical model of herding (e.g., Welch (1992), Bikhchandani et al. (1992), and Banerjee (1992)), each player's payoff is independent of the actions by other players. In our model, however, each voter's payoff depends on other voters' behavior because it affects the voting outcome. Callander (2007) and Ali and Kartik (2012), however, are exceptions because they study herding in the context of sequential voting.[7] One difference between their work and ours is the focus: they assume two signals where each signal implies one of the two states is more likely. In contrast, since we are

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[5]They also provide one example with three signals to argue that sequential voting can perform better than simultaneous voting.

[6]See also their extension to the three-option case with a choice of the timing of moves (Dekel and Piccione, 2014).

[7]Ali and Kartik (2012) consider a general setting that applies not only to voting but to other contexts as well.

interested in the implication of the differentiated precision of signals, we have three signals to include the possibility of uninformed voters as in Feddersen and Pesendorfer (1996).[8]

Let us proceed with more detailed comparisons. Callander (2007) postulates the existence of "conformity" incentives (the desire to vote for the majority) and demonstrates the potential emergence of herding, or bandwagon, in equilibrium. Conversely, we do not assume conformity, and introducing such an incentive would change the result.[9] It is noteworthy that his model, too, captures the tradeoff between the posterior effect and the delegation effect. Specifically, one of his analyses considers a strategy profile that uses a "bandwagon threshold." This threshold is defined by the vote lead below which voters follow their own signal and above which they herd. He shows that, absent the conformity incentive, a voter situated just preceding the bandwagon threshold would have a strict incentive to vote against the majority due to the incentive similar to the delegation effect, assuming all other voters follow the threshold strategy. In contrast, the pivotal voter in our morel would be indifferent between following the majority and delegating. This disparity arises because, in his model, after a pivotal voter has voted against the majority, the next voter is not pivotal as the bandwagon threshold only hinges on the vote lead. The dependence of the bandwagon threshold on the vote lead is intrinsic to his infinite population model and indeed arises in equilibrium when the voters have conformity incentives. Conversely, in our finite population model, the subsequent voter retains the pivotal role. This difference implies that voters in Callander (2007)'s infinite population model and those in our finite population model undertake markedly distinct calculations in their voting decisions. We note that, due to the strict incentive to vote against the majority in the case of zero conformity incentive, the only possible equilibrium in the class Callander (2007) analyzes is the one where the bandwagon threshold is infinity, that is, every voter follows their own signal (his Theorem 5). In contrast, we have a wide range of equilibria while not requiring conformity incentives.

The study by Ali and Kartik (2012) centers around a particular strategy termed

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[8]Although a two-signal setup is typical in the papers on sequential voting, some papers have considered the case in which different voters receive signals of different precision. For example, Ottaviani and Sørensen (2001) and Alpern and Chen (2022) considered such a setting. Their main interest, however, is in the optimal ordering of voters, and our model has different model specifications from theirs (voters have reputation concerns, signal structures are different, etc.).

[9]Indeed, in the Online Appendix, we consider a version of an incentive to end the game and show that long voting occurs in the unique equilibrium.

"sincere behavior," wherein voters base their choices only on the posterior belief about the likelihood of each state, independently of the consideration of the event under which they are pivotal, or the impact of their votes on subsequent participants. Their main result establishes that sincere behavior constitutes an equilibrium despite the potential tradeoff between the posterior and delegation effects. Although this result is intriguing, the restriction to sincere behavior imposes a somewhat severe constraint on the possible equilibrium outcome.[10] In contrast, our "anything goes" result shows that a wide range of behavior can be consistent with equilibrium behavior. This discrepancy comes from both the focus on sincere behavior as well as the difference in the signal structure: adapting sincere behavior to our model would only predict a rather small set of equilibria, and an "anything goes" result is even difficult to define in their two-signal model.[11]

Although the herding effect highlights the failure of sequential voting as an information aggregation device, some articles have shown that sequential voting facilitates learning from early votes, which cannot happen under simultaneous voting (e.g., Dekel and Piccione (2000), Gershkov and Szentes (2009), Selman et al. (2010), Hummel and Knight (2015) and Alpern and Chen (2022)). The theory of information aggregation under sequential voting is supported by experimental work as well (e.g., Battaglini et al. (2007)). Our results on welfare are similar to the ones in these papers because we show that consistent equiliria in our sequential model outperform the symmetric equilibrium of the corresponding simultaenous-voting game, and inconsistent equilibria can yield the upper bound of the payoff that can be achieved by any mechanism. However, we also show that inconsistent equilibria can yield a payoff lower than the one in the simultaneous-voting case.

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[10]For example, regardless of the signal structure in the class their model permits, if there are three consecutive votes on an option after any history involving two consecutive votes on the other option, their equilibrium predicts that the voters must herd on the former option with probability one. The proof of this result is available upon request.

[11]In our model, sincere behavior would result in a consistent strategy equilibrium where the default sequence is either "all voters vote for A" or "all voters vote for B." Our "anything goes" result pertains to a possible case in which no one receives an informative signal, but this situation never happens in their two-signal model since every voter is assumed to receive some informative signal.

2 Model

There are voters $1, 2, \ldots, 2n + 1$, where n is a non-negative integer.[12] The voters sequentially vote on either A or B. We assume the majority voting rule: A results as soon as $n + 1$ voters vote for A, and B results as soon as $n + 1$ voters vote for B. Formally, let the set of *sequences* be

$$\mathcal{S} := \bigcup_{k=n+1}^{2n+1} \{(X_1, \ldots, X_k) \in \{A, B\}^k \mid |\{l \in \{1, 2, \ldots, k\} \mid X_l = X_k\}| = n + 1\}.$$

That is, a sequence is a path of actions that ends the game. For any $S = (X_1, \ldots, X_k) \in \mathcal{S}$, call X_k the *outcome* of S.

The state of the world is either α or β. The prior probability for each state is $1/2$. When the state is α, each voter, when she is called upon to vote, independently receives signal a with probability $\varepsilon > 0$; otherwise (i.e., with probability $1 - \varepsilon$), she receives no signal, or formally we say she receives signal $\emptyset$. Similarly, when the state is β, each voter, when voting, independently receives signal b with probability ε and signal $\emptyset$ with probability $1 - \varepsilon$. Given this simple information structure, signals a and b perfectly reveal the state of the world.

All voters receive the payoff of 1 if the outcome of the realized sequence coincides with the state, i.e., the outcome is A when the state is α or it is B when the state is β; otherwise, all voters receive the payoff of 0. Note that the model is characterized by parameters (n, ε).

For voter k, a *history* is an enumeration of actions by $k - 1$ voters before k, and her own signal. Thus, the set of voter k's histories is

$$H_k = \{A, B\}^{k-1} \times \{a, b, \emptyset\}.$$

Voter k's mixed strategy is a mapping $\sigma_k : H_k \to \Delta(\{A, B\})$. Let Σ_k be the set of all strategies of voter k and let $\Sigma = \times_{k=1}^{2n+1} \Sigma_k$.

In this paper, we consider the concept of sequential equilibrium. We sometimes refer to it as "equilibrium" for short.

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[12]We use $2n + 1$ as the number of voters instead of, say, N, which we indeed use later in a more general setting when the number of voters need not be odd, not just to emphasize that there are odd number of voters but because the number n itself will be a key variable. For example, we will pay special attention to the case where there have been exactly n votes for one of the two options.

We say that a strategy σ_k is *consistent* if $\sigma_k(\cdot, a)(A) = 1$ and $\sigma_k(\cdot, b)(B) = 1$. Although consistency may be a natural property, it will turn out that there are equilibria that are not consistent and, in fact, they may have a greater expected payoff than any equilibria that are consistent.

Given any pure strategy profile $\sigma \in \Sigma$, there is a unique sequence S that realizes if no voter receives a signal. We call such S the *default sequence of σ*.

3   Consistent Strategies

Our first set of results pertains to equilibria that use consistent strategies. Section 3.1 shows that any sequence can be a default sequence of a consistent equilibrium, and Section 3.2 shows that any such equilibria have the same welfare.

3.1   Anything Goes

Our first result is rather striking. It says: anything goes!

Theorem 1. *For any $n \geq 0$, $\varepsilon \in (0,1]$ and any sequence $S \in \mathcal{S}$, there is a pure consistent strategy equilibrium σ in the model with (n, ε) such that S is the default sequence of σ.*

That is, any path of actions can be a default sequence of an equilibrium. This in particular implies that paths in which the voting ends early (e.g., $n + 1$ straight As) can also realize in equilibrium. This conclusion contrasts with the intuition of Feddersen and Pesendorfer (1996) that a voter who does not have information would like to leave the decision to other voters.

The proof constructs an equilibrium for each S. A rough intuition for the construction and why it works is as follows. If voter k without information is supposed to play A in the default sequence S but plays B, since such a deviation can only be caused when k has signal b, the subsequent voters with no information will rationally update their belief to assign probability 1 to state β. Moreover, even in the case when some of the subsequent voters receive signal a and play A, we let the voters without information have a belief that assigns probability 1 to β and play B.[13] This will make it hard for the subsequent voters to use the information they receive. Hence, voter

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[13]Such a belief can be shown to satisfy consistency of Kreps and Wilson (1982).

k has an incentive to follow the default sequence when she does not receive a signal. We denote the strategy profile constructed in this way by σ^S.

When we presented this result in seminars, one common reaction was that the result would be trivial because the voter with no information assigns equal probability to each state so must be indifferent, and this should be why "anything can happen." This argument misses the point that, in consistent equilibria, each voter's action changes the belief of the subsequent voters. Thus, each voter's best response depends on the action sequence by the preceding voters (which affects her posterior belief) as well as on the effect of her action on the subsequent voters' beliefs. Indeed, the voters do not necessarily have posterior beliefs that assign equal probability to each state, and thus they are not necessarily indifferent. More precisely, on the path of play of any pure consistent equilibrium, a voter is indifferent if and only if she is *deciding*: Under σ^S, say that voter k is *deciding* if when the history up to k has been consistent with S, if no voter in $\{1, \ldots, 2n+1\}$ receives a signal, the outcome varies by k's vote, and one of k's vote immediately ends the game. For example, when $n = 2$, if $S = (A, B, A, B, A)$, then only $k = 5$ is deciding. Note that $k = 4$ is not deciding because the outcome is A irrespective of his action (given that no one receives a signal and everyone else follows σ^S). If $S = (A, A, B, B, B)$, then $k = 3, 4, 5$ are all deciding.

Now, consider voter $k < 2n + 1$ who is deciding received no signal, and suppose, without loss of generality, that the game ends if she plays A.[14] Since k is deciding, if she plays B and no subsequent voter receives a signal, then all the subsequent voters will play B and thus the last voter $2n+1$ gets to play. The proof of Theorem 1 shows that voter k is indifferent between playing A and B under this situation. In what follows, we provide three explanations for why this is true.

The first explanation is due to the comparison of the benefit of deciding on the currently superior outcome and postponing the decision to benefit from the subsequent voters' information: Voter k assigns a high posterior on the state being α because A has been played n times while B has been played less than n times. This makes playing A attractive for voter k. We call this effect the *posterior effect*. On the other hand, passing the decision to the subsequent voters makes it possible to use the information obtained by those voters. This makes playing B attractive. This intuition is analogous to the one in Feddersen and Pesendorfer (1996), and we call

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[14]If $k = 2n + 1$, then k is indifferent because the posterior assigns equal probability to each state and k's vote determines the outcome.

this effect the *delegation effect*.

Why do these two effects cancel each other? If k plays A, then she receives the payoff of 1 with the probability that is the posterior belief on α, which we denote by $P(\alpha)$. If k were to be able to end the game with outcome B, then her payoff would be $1 - P(\alpha)$. Noting that $P(\alpha) > 1/2 > 1 - P(\alpha)$ as we discussed, the benefit from playing A due to the higher posterior on α can be expressed as $P(\alpha) - (1 - P(\alpha))$. If instead she plays B, then voter k gets payoff 1 if and only if the game ends with outcome A when the state is α and at least one of the subsequent voters receives a signal, or when the state is β. Thus, the benefit from playing B due to the information that a subsequent voter may receive can be expressed as $P(\alpha) \cdot (1 - (1 - \varepsilon)^{2n+1-k})$, which is equal to $P(\alpha) - P(\alpha)(1-\varepsilon)^{2n+1-k}$. Now, since $P(\alpha) = \frac{(1-\varepsilon)^{k-n}}{(1-\varepsilon)^{k-n}+(1-\varepsilon)^{n+1}}$ and $1 - P(\alpha) = \frac{(1-\varepsilon)^{n+1}}{(1-\varepsilon)^{k-n}+(1-\varepsilon)^{n+1}}$, we have:[15]

$$1 - P(\alpha) = P(\alpha)(1-\varepsilon)^{2n+1-k}.$$

This is why the two effects cancel each other.

The second explanation is based on a state-by-state comparison: If the state is α, with probability $(1-\varepsilon)^{2n+1-k}$, playing B decreases the payoff by 1 (relative to playing A). If the state is β, with probability 1, playing B increases the payoff by 1 (again, relative to playing A). The ratio of the posteriors on the two states is $(1-\varepsilon)^{k-n}$ to $(1-\varepsilon)^{n+1}$, that is, 1 to $(1-\varepsilon)^{2n+1-k}$.

The third explanation is due to the consideration of the event in which voter k becomes pivotal: If k chooses A, then that ends the game. If she chooses B, then if at least one of the subsequent voters plays A then the outcome is A. If all the subsequent voters play B, then that means that (i) n other voters played A when they were supposed to play A given no signal, and (ii) n other voters played B when they were supposed to play B given no signal. Hence, conditional on this event, voter k is indifferent between A and B. Hence, her expected payoff from playing B would not change even if we modified the game and hypothetically assigned outcome A to this event. With this modification, the outcome from k's playing B is always A.

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[15] The posterior $P(\alpha)$ can be computed as follows. When the state is α, The history so far follows the default sequence when no voter who was supposed to vote for B on the default sequence received a signal. Hence, the probability that the state is α, the history so far follows the default sequence, and k does not receive a signal is $\pi_A := \frac{1}{2}(1-\varepsilon)^{k-1-n}(1-\varepsilon)$. Similarly, the probability that the state is β, the history so far follows the default sequence, and k does not receive a signal is computed to be $\pi_B := \frac{1}{2}(1-\varepsilon)^n(1-\varepsilon)$. We use these values to compute the posterior $P(\alpha) = \frac{\pi_A}{\pi_A+\pi_B}$.

11

Thus, voter k must be indifferent between playing A and playing B.

Let us formally state the result about when a voter is indifferent.

Remark 1. Fix (n, ε). For any $S \in \mathcal{S}$, consider σ^S and voter k who observed a history that does not contradict S. When voter k does not receive a signal, she is indifferent between voting for A and B if and only if k is deciding.

One may wonder our equilibrium construction may hinge "too much" on the observation of the particular default sequence. However, the "anything goes" result continues to hold even if we allow voters to observe only the vote counts, where we modify our equilibrium construction. We present this result in Section 6.4.

Finally, we note that the result depends on the symmetry of the model. However, we still believe the "anything goes" result is useful for two reasons. First, in Section 6.1, we examine the case with asymmetry in the prior, the signal probabilities, the voting rule, and the payoffs. We show that if the asymmetry is introduced in a way that "favors" a particular outcome (A or B), then any sequence that induces the favored outcome is a default sequence of some pure consistent equilibrium. Thus, "anything goes" is still partially true with a restriction on the outcome. In particular, even with asymmetry, both long voting and short voting can arise in equilibrium. Second, as in our explanation after its statement, the "anything goes" theorem gives us a way to understand the incentives faced by the voters in the most simplified setting. The posterior effect and the delegation effect will help us understand why a particular asymmetry would imply the partial "anything goes" result that we present in Section 6.1.

3.2   Welfare Invariance

Take an arbitrary pure consistent equilibrium σ. Suppose without loss of generality that σ induces the outcome of A with probability 1 if no voter receives a signal. If the state is α, then no one receives signal b and thus those who are supposed to play A on the default sequence of σ would never switch their actions to B. Consequently, the outcome becomes A with probability 1. On the other hand, if the state is β, there are $n+1$ chances to overturn the outcome. This occurs if some voter who is supposed to play A on the default sequence of σ receives signal b and thus plays B. Note that if such an event occurs, then by the Bayes rule, all the subsequent voters must have a belief that assigns probability 1 to β, and thus the outcome will be B. Therefore,

the outcome becomes B with probability $1 - (1 - \varepsilon)^{n+1}$ for any such σ. Overall, the ex ante payoff under σ is

$$\frac{1}{2} \cdot 1 + \frac{1}{2}(1 - (1 - \varepsilon)^{n+1}) = 1 - \frac{(1 - \varepsilon)^{n+1}}{2}.$$

Thus, we have shown the following.

Theorem 2. *For any* (n, ε), *pure consistent strategy equilibrium in the model with* (n, ε) *has the ex ante payoff of* $1 - \frac{(1-\varepsilon)^{n+1}}{2}$.

This welfare invariance can be suitably extended to the settings with asymmetry (Corollary 2 in Section 6.1): all the pure consistent equilibria have the same payoff.

4   Inconsistent Strategies

So far, we have restricted our analysis to equilibria that use consistent strategies. Perhaps surprisingly, there exist equilibria that use inconsistent strategies. This section is devoted to an analysis of those equilibria.

Before proceeding, let us provide one example to illustrate why the existence of such equilibria is not trivial. Suppose there are at least three voters $(n \geq 1)$ and consider the strategy profile in which all voters vote for A irrespective of their information. This strategy profile is an equilibrium in the simultaneous voting model because each voter is indifferent as their vote would not affect the outcome. This strategy profile, however, does not constitute an equilibrium in our sequential voting model. The reason is that, for example, the last voter (voter $2n+1$) would vote for B if she obtained a chance to vote and received signal b. Furthermore, expecting this, preceding voters who faced the history of n times of A and received signal b would vote for B as well.

Theorem 2 shows that all consistent equilibria have the same expected payoff. We show that inconsistent equilibria have a wide range of expected payoffs. Section 4.1 provides a payoff upper bound under inconsistent equilibria. Such a bound can be obtained by using strategies that let the voters vote against their signals. Section 4.2, in contrast, provides a payoff lower bound. This bound is obtained by using strategies that let the voters vote independently of their signals.

13

4.1 Payoff Upper Bound: Voting against a Signal Improves Welfare

This section is devoted to the analysis of the payoff upper bound. The bound can be obtained by using strategies that let the voters vote for B given signal a and for A given signal b. The following example shows that there exists an equilibrium that uses such inconsistent strategies and achieves a strictly larger expected payoff than the one under any consistent equilibrium.

Example 1. Assume there are 3 voters. The following strategy profile constitutes an equilibrium, which we denote by $\bar{\sigma}$.

- Voter 1 chooses B if she receives a; otherwise, she plays A.

- Voter 2 uses a consistent strategy that plays the opposite action to voter 1's if he receives no signal.

- Voter 3 uses a consistent strategy that plays the same action as voter 2's if she receives no signal.

Note that the default sequence of this strategy profile $\bar{\sigma}$ is (A, B, B). Here we explain that voter 1 has a strict incentive to vote against her signal under $\bar{\sigma}$.[16] Consider voter 1. Suppose she receives signal a. This happens only if the state is α, and hence, the subsequent voters receive either a or ϕ. Therefore, if voter 1 follows the above strategy by choosing B, her expected payoff becomes 1 since voters 2 and 3 play A with probability 1. If she instead plays A, then her payoff is $1 - (1 - \varepsilon)^2$. Hence, she strictly prefers to play B. Suppose voter 1 receives signal b. If she plays A, her payoff is 1. If she plays B, her payoff is $1 - (1 - \varepsilon)^2$. Hence, she strictly prefers to play A.

We compute the ex ante expected payoff from $\bar{\sigma}$. First, if the state is β, then the outcome is B with probability 1. Second, if the state is α, then outcome A realizes if at least one voter out of the three voters receives signal a. Thus, the payoff is

$$\frac{1}{2} \cdot 1 + \frac{1}{2}[1 - (1 - \varepsilon)^3].$$

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[16]The proof of Theorem 3 fully checks the voters' incentives to show that this strategy profile is indeed an equilibrium.

Note that the payoff from any pure consistent equilibrium is

$$\frac{1}{2} \cdot 1 + \frac{1}{2}[1 - (1 - \varepsilon)^2].$$

Hence, the payoff is strictly greater under $\bar{\sigma}$ than in any pure consistent equilibrium.

The intuition is as follows. Under $\bar{\sigma}$, the default sequence leads to outcome B, so voters receiving signal b would not be incentivized to vote against the default sequence. In contrast, we would like to maximize the chances for voters receiving signal a to overturn the outcome to A. In the current example, there are at most three such chances (since there are three voters) and $\bar{\sigma}$ uses all these three chances. This is made possible because, when a voter is supposed to vote for A in the default sequence and receives signal b, we have her vote for A (as for voter 1 in the current example). This is incentive compatible because, as we mentioned, the voter expects that the outcome will be B when she follows the default sequence (i.e., she votes for A). Therefore, her voting for B can be used as a message to the subsequent voters that she has received signal a (not signal b!). This construction is different from the one for consistent equilibria where any voter receiving signal b votes for B. □

The equilibrium construction in the above example can be generalized. The next theorem formalizes this point. Let $\bar{R}_n := 1 - \frac{(1-\varepsilon)^{2n+1}}{2}$. Note that when $n = 1$, $\bar{R}_n$ is the equilibrium payoff we derived in the above example.

Theorem 3. *For any n, there is an equilibrium σ that achieves the payoff $\bar{R}_n$.*

The proof in the Appendix shows that a strategy profile generalizing $\bar{\sigma}$ in Example 1 is an equilibrium.

We now show that $\bar{R}_n$ is the highest feasible payoff that can be achieved. To formalize this statement, given n, consider a social choice function $f : \{a, b, \emptyset\}^{2n+1} \to \Delta(\{A, B\})$ that returns a probability distribution over outcomes for each realization of signals of all the $2n + 1$ voters. Note that for any strategy profile σ in our original game, there is a social choice function f that achieves the same distribution over outcomes conditional on any realization of the state and the signals. Let F_n be the space of all social choice functions. Notice that a social choice function $f \in F_n$ determines the ex ante expected payoff to each voter. Let this payoff be $R_n(f)$.

Proposition 1. *For all $f \in F_n, R_n(f) \le \bar{R}_n$.*

15

Proof. Fix any $f \in F_n$. Let $f(\emptyset, \ldots, \emptyset)(A) = q$. Then, conditional on the signal realization $(\emptyset, \ldots, \emptyset)$, the voters' expected payoff is $\frac{1}{2}q + \frac{1}{2}(1-q) = \frac{1}{2}$. The probability that the signal profile $(\emptyset, \ldots, \emptyset)$ realizes is $(1-\varepsilon)^{2n+1}$. Hence, the ex ante expected payoff is at most $(1 - (1-\varepsilon)^{2n+1}) \cdot 1 + (1-\varepsilon)^{2n+1} \cdot \frac{1}{2} = 1 - (1-\varepsilon)^{2n+1} \cdot \frac{1}{2} = \bar{R}_n$. $\quad\square$

We note that the payoff $\bar{R}_n$ can be realized by a social choice function that, whenever there is at least one signal, the outcome accords with that signal (and returns an arbitrary outcome when there is no signal). The intuition is that the social choice function asks every voter if they have received a signal and returns the outcome according to the voter's response. What Theorem 3 shows is that we can construct an equilibrium corresponding to such a social choice function. In contrast, Theorem 2 shows that the consistent strategy equilibria correspond to asking only $n + 1$ voters about their signals.

4.2 Payoff Lower Bound: Ignoring a Signal Leads to Inefficiency

Now we turn to the analysis of the payoff lower bound. The bound can be obtained by using a strategy that lets the voter vote for a particular action regardless of the signal. The following example shows that there exists an equilibrium that uses such inconsistent strategies and achieves a strictly lower expected payoff than that of pure consistent equilibria.

Example 2. Consider the case with 5 voters and the following strategy profile, which we denote by $\underline{\sigma}$.

- Voters $k = 1, 2$: Voter 1 plays A irrespective of the signal. Voter 2 plays A irrespective of the signal if voter 1 has played A. Otherwise, he plays a consistent strategy in which he plays A if no signal is received.

- Voters $k = 3, 4$: If all the actions observed so far are 2 times of A followed by $k - 3$ times of B, then play B irrespective of the signal. Otherwise, play a consistent strategy in which she plays A if no signal is received.

- Voter $k = 5$: Play a consistent strategy in which she plays A if no signal is received.

The default sequence of this strategy profile $\underline{\sigma}$ is (A, A, B, B, A). Note that $\underline{\sigma}$ has the ex ante payoff of $\frac{1}{2} + \frac{1}{2}\varepsilon = \frac{1+\varepsilon}{2}$. We can show that when $\varepsilon > 0$ is sufficiently small, $\underline{\sigma}$ constitutes an equilibrium with the belief that, after any deviation, any voter who has not received a signal assigns probability 1 to α. Here, let us explain why voters have incentives to ignore the signals on the path of play.[17]

First, voters 1 and 2 play A even if they receive signal b. The reason is that if they play B, then the subsequent voters believe that the state is α. For the outcome to be B under such a situation, there need to be at least two subsequent voters who receive signal b, which happens with probability $O(\varepsilon^2)$.[18] In contrast, if they follow $\underline{\sigma}$ and play A, then the outcome will be B if and only if voter 5 receives signal b, which happens with probability ε. Thus, it is indeed optimal to ignore the signal and play A when $\varepsilon > 0$ is sufficiently small.

Second, voters 3 and 4 play B even if they receive signal a. The reason is that even if they do so, the outcome will be A with probability 1 according to $\underline{\sigma}$ given that the state is α, which means that playing B ensures that the payoff is 1. Thus, ignoring the signal and playing B is indeed optimal. $\qquad\square$

The equilibrium construction in the above example can be generalized. The next theorem formalizes this point. Recall that the equilibrium constructed in the above example achieves the payoff of $\frac{1+\varepsilon}{2}$.

Theorem 4. *For any $n \geq 2$, there is $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$, there is an equilibrium that achieves the payoff of $\frac{1+\varepsilon}{2}$.*

The proof in the Appendix shows that a strategy profile generalizing $\underline{\sigma}$ in Example 2 is an equilibrium.

We now show that the payoff identified in Theorem 4 is the worst possible payoff. The meaning of "worst" needs an explanation because we do not consider all social choice functions. For example, imagine social choice functions that always return the

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[17]The proof of Theorem 4 fully checks the voters' incentives to show that this strategy profile is indeed an equilibrium.

[18]We note that the argument depends on $\varepsilon > 0$ being small. To see this, consider voter 2 who observes voter 1's A and receives signal b. If he plays A, then the outcome will be B if and only if voter 5 receives signal b, which happens with probability ε. Hence, his payoff in this case is ε. If he plays B instead, then the outcome will be B if and only if two or three out of three subsequent voters receive signal b. The probability of this event is $3\varepsilon^2(1-\varepsilon) + \varepsilon^3$, and hence his payoff in this case is $3\varepsilon^2(1-\varepsilon) + \varepsilon^3$. The latter payoff is greater than the former if and only if $\varepsilon < 3\varepsilon^2(1-\varepsilon) + \varepsilon^3$, or $(2\varepsilon - 1)(\varepsilon - 1) < 0$, which holds if and only if $\varepsilon > \frac{1}{2}$.

outcome that is against the signals obtained by the voters. Such social choice functions induce a payoff that is obviously strictly below the payoff that any equilibrium of our sequential voting game can achieve. For this reason, we seek to obtain the lowest possible pure strategy equilibrium payoff that can be achieved in our sequential voting game.

Proposition 2. *Given any (n, ε), the expected payoff from any pure strategy equilibrium is at least $\frac{1+\varepsilon}{2}$.*

The proof is involved. We show that the last voter on the default sequence uses a consistent strategy[19], and use this property to bound the payoffs of the previous voters inductively. We note that the payoff $\frac{1+\varepsilon}{2}$ corresponds to the payoff from the social choice function that asks only one voter about her signal and returns the outcome that accords with her signal if she receives one. This payoff may appear quite low, but it is strictly greater than an equilibrium payoff in the simultaneous-move version of the game: If $n > 1$ (i.e., there are 3 or more voters), the simultaneous-move voting has an equilibrium in which every voter votes for A regardless of the signal, and this equilibrium has the payoff of $\frac{1}{2}$.

Let us summarize this section. There are various kinds of inconsistent strategy equilibria, and their welfare ranges from the best payoff achievable under any social choice function, which corresponds to asking all the $2n + 1$ voters about their signals, to the one that corresponds to asking just a single voter about her signal. The payoff under the consistent equilibria corresponds to asking $n + 1$ voters about their signals and thus positions between those two bounds.

5   Long Voting

A simple application of the Feddersen and Pesendorfer (1996)'s intuition might lead one to conjecture long voting, but the "anything goes" result shows that long voting is only one of many possible predictions. In this section, we consider two modification of our model that leads to the unique prediction of long voting.

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[19]This means that we need to rule out the strategy of, for example, playing A to end the game given no signal and playing B to continue the game given signal a or b.

5.1   When the Last Voter "Trembles"

Consider a modified game in which voter $2n + 1$ is constrained to choose a mixed action (he trembles, as in the last voter in *Squid Game*). Formally, for any $\xi \in (0, \frac{1}{2}]$, a ξ-*tremble game* is a game that is the same as the one in the main section, except that voter $2n + 1$, upon moving, chooses a probability distribution over $\{A, B\}$ such that each action receives probability at least ξ if he does not receive a signal. We say that a strategy profile in a ξ-tremble game is *pseudo-pure* if every voter from 1 to $2n$ uses a pure strategy.

Theorem 5. *In any ξ-tremble game, there is no consistent pseudo-pure strategy equilibrium such that the game ends before reaching voter $2n + 1$ when no voter receives a signal.*

The intuition for the proof is as follows. Consider voter k who is deciding (see the discussion after Theorem 1 for the definition of "deciding"), and suppose that this voter is not the last voter in the game (i.e., $k < 2n + 1$). Suppose without loss of generality that the game ends if she plays A. We know that the posterior effect and the delegation effect make actions A and B attractive, respectively, and consequently, k is indifferent between playing A and B. This is based on the fact that all the subsequent voters receiving no signal will play B after k plays B. However, if there is a chance that, after k's playing B, a subsequent voter receiving no signal plays A, then voter k is no longer indifferent between the two actions. This is because, since state α is more likely than β, the posterior effect makes playing B more attractive as well. This is why it is impossible to sustain k's playing A as an equilibrium action.

In Appendix A.5, we prove this theorem as a corollary of a more general result: we suppose that one voter k^* (not necessarily the last voter) trembles and show that, in equilibrium, the game does not end before k^* if no voter receives a signal.

We note that "long voting" means that the voting takes a long time if no voter receives a signal. If some voter receives a signal and votes against the default sequence, the voting can end early. This means that, in the environment where the last voter trembles, shorter voting is associated with a higher expected payoff.[20]

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[20] An analogous comment applies to the modification of the model in the next subsection.

5.2 Small Incentive Not to End

Consider a model in which we change the payoff function of each voter so that if voter k votes for X when the history has n times of X, then k incurs a positive cost $c > 0$. Call this game the *pivot-aversion game*. Pivot aversion seems natural in situations where voters dislike being considered at their fault about the realized outcome.

Theorem 6. *In any pivot-aversion game, there is no pure consistent strategy equilibrium such that the game ends before reaching voter $2n + 1$ when no voter receives a signal.*

The intuition is simple. In the original voting game, as long as voters play a pure consistent equilibrium, any voter who is deciding is indifferent due to the posterior effect and the delegation effect (see Remark 1). In the pivot-aversion game, such a voter incurs an additional cost from ending the game. Hence, the indifference is broken in favor of the delegation effect, and thus she votes for an option that makes the voting continue. This is why the game lasts until it reaches the last voter.

Although pivot aversion seems natural, one may argue that it is an ad hoc assumption to obtain the long voting result. In the Online Appendix, we consider a version of incentives in which voters want to end the game and show a long voting result.

6 Discussions

This section provides several discussions. Section 6.1 introduces asymmetries to our base model and examines how our "anything goes" result and the welfare invariance are affected. In Section 6.2, we compare our consistent equilibria with an equilibrium in the static version of the voting game. Section 6.3 discusses a model with abstention, and Section 6.4 considers a restriction to strategies that depend only on the vote count to date.

6.1 General Asymmetric Voting

In this section, we explore how the result in Theorem 1 is affected by an introduction of asymmetry in the environment. For this purpose, we modify the model as follows:

1. The prior probability on α is $p \in (0, 1)$.

2. The probability of receiving signal a at state α is $\varepsilon_a \in (0, 1)$ and that of receiving signal b at state β is $\varepsilon_b \in (0, 1)$.

3. There are N voters. K votes are necessary to achieve outcome A and L votes are necessary to achieve outcome B, where $K + L - 1 = N$.

4. All voters receive the payoff of $\gamma_\alpha > 0$ if the outcome is A when the state is α and $\gamma_\beta > 0$ if the outcome is B when the state is β; otherwise, all voters receive the payoff of 0.

We call this model a *general voting model with* $(p, \varepsilon_a, \varepsilon_b, K, L, \gamma_\alpha, \gamma_\beta)$. Our model in Section 2 is a general voting model with $(\frac{1}{2}, \varepsilon, \varepsilon, n + 1, n + 1, 1, 1)$.

We extend the definition of sequences to this general environment by letting the set of *sequences* be

$$
\mathcal{S} := \left(\bigcup_{k=K}^{N} \{(X_1, \ldots, X_k) \in \{A, B\}^k \mid |\{l \in \{1, 2, \ldots, k\} | X_l = A\}| = K, X_k = A\} \right) \cup
$$

$$
\left(\bigcup_{k=L}^{N} \{(X_1, \ldots, X_k) \in \{A, B\}^k \mid |\{l \in \{1, 2, \ldots, k\} | X_l = B\}| = L, X_k = B\} \right).
$$

As before, for any $S = (X_1, \ldots, X_k) \in \mathcal{S}$, call X_k the *outcome* of S.

Proposition 3. *Consider the general voting model with* $(p, \varepsilon_a, \varepsilon_b, K, L, \gamma_\alpha, \gamma_\beta)$ *and fix a sequence* $S \in \mathcal{S}$. *Then, the following holds.*

1. *If* $\frac{1-p}{p} \frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L} \frac{\gamma_\beta}{\gamma_\alpha} < 1$, *then there is a pure consistent strategy equilibrium* σ *such that* S *is the default sequence of* σ *if and only if the outcome of* S *is* A.

2. *If* $\frac{1-p}{p} \frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L} \frac{\gamma_\beta}{\gamma_\alpha} = 1$, *then there is a pure consistent strategy equilibrium* σ *such that* S *is the default sequence of* σ.

3. *If If* $\frac{1-p}{p} \frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L} \frac{\gamma_\beta}{\gamma_\alpha} > 1$, *then there is a pure consistent strategy equilibrium* σ *such that* S *is the default sequence of* σ *if and only if the outcome of* S *is* B.

That is, although the theorem shows that introducing asymmetry breaks the "anything goes" result, it identifies the direction in which it is broken. Moreover, it shows that any sequence, whether long or short, that leads to a particular outcome can be a default sequence of a pure consistent strategy equilibrium.

The following corollary provides some special cases that are helpful in understanding this general result.

Corollary 1. *Consider the general voting model with $(p, \varepsilon_a, \varepsilon_b, K, L, \gamma_\alpha, \gamma_\beta)$ and suppose that one of the following holds:*

1. $p < \frac{1}{2}$, $\varepsilon_a = \varepsilon_b$, $K = L$, and $\gamma_\alpha = \gamma_\beta$.

2. $p = \frac{1}{2}$, $\varepsilon_a > \varepsilon_b$, $K = L$, and $\gamma_\alpha = \gamma_\beta$.

3. $p = \frac{1}{2}$, $\varepsilon_a = \varepsilon_b$, $K < L$, and $\gamma_\alpha = \gamma_\beta$.

4. $p = \frac{1}{2}$, $\varepsilon_a = \varepsilon_b$, $K = L$, and $\gamma_\alpha < \gamma_\beta$.

Then, there is a pure consistent strategy equilibrium σ such that S is the default sequence of σ if and only if the outcome of S is B.

On the one hand, the "if" direction of the proof of the proposition (that is, to show that any sequence with outcome B can be a default sequence) closely follows that of Theorem 1.

On the other hand, the "only if" direction of the proof of the proposition (that is, to show that no sequence with outcome A can be a default sequence) closely follows that of Theorem 5 because the proof is about breaking the indifference due to the balance between the posterior effect and the delegation effect. To understand this point, consider a pure consistent strategy profile in which the default sequence is the K times of A. When $p = \frac{1}{2}$, $K = L(= n + 1)$ and $\varepsilon_a = \varepsilon_b$, voter $n + 1$ was indifferent between playing A and B due to the balance of the two effects.

Now, consider the three settings in Corollary 1 and suppose for simplicity that the first $K - 1$ voters have voted for A and it is voter K's turn. This voter would be indifferent between voting for A and B in the perfectly symmetric model. In part 1 ($p < \frac{1}{2}$), the posterior effect is undermined, and hence there is a strict incentive to vote for B. In part 2 ($\varepsilon_a > \varepsilon_b$), since signal a is more likely than signal b, after $K - 1(= L - 1)$ times of straight As, the posterior effect is not large (as B has been relatively unlikely to be chosen under state β), and the delegation effect is strong (as A will relatively likely be chosen under state α). In part 3 ($K < L$), the posterior effect is given by the $K - 1$ times of A, while the delegation effect comes from $L - 1$ times of B. Hence, the delegation effect is stronger, and voter K has a strict incentive to vote for B. In part 4 ($\gamma_\alpha < \gamma_\beta$), since the posterior effect and the delegation effect

22

are the same as in the symmetric model, the payoff from ending the vote with each outcome determines the incentive. Since $\gamma_\alpha < \gamma_\beta$, voter K has a strict incentive to vote for B.

To summarize, if the asymmetry is introduced in a way that "favors" a particular outcome in light of the balance between the posterior effect and the delegation effect or the payoff from each outcome, then no sequence that induces the non-favored outcome can be a default sequence of any pure consistent equilibrium, while any sequence that induces the favored outcome is a default sequence of some pure consistent equilibrium. In this sense, the result shows a limitation as well as the robustness of the "anything goes" result. In particular, it shows that both long voting and short voting can be an equilibrium default sequence, even under asymmetry.

Given Proposition 3, the welfare equivalence result (Theorem 2) can be extended as follows:

Corollary 2. *Consider the general voting model with $(p, \varepsilon_a, \varepsilon_b, K, L, \gamma_\alpha, \gamma_\beta)$. The voters' payoff from any pure consistent strategy equilibrium is*

$$\begin{cases} p\gamma_\alpha + (1-p)(1-(1-\varepsilon_b)^K)\gamma_\beta & if \quad \frac{1-p}{p}\frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L}\frac{\gamma_\beta}{\gamma_\alpha} < 1 \\ p(1-(1-\varepsilon_a)^L)\gamma_\alpha + (1-p)\gamma_\beta & if \quad \frac{1-p}{p}\frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L}\frac{\gamma_\beta}{\gamma_\alpha} \geq 1 \end{cases}.$$

To understand this corollary, consider the case of $\frac{1-p}{p}\frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L}\frac{\gamma_\beta}{\gamma_\alpha} < 1$. Then, Proposition 3 implies that the outcome of the default sequence of any pure consistent strategy equilibrium is A. Hence, whenever the state is α (which happens with probability p), the voters receive the payoff of α. If, on the other hand, the state is β (which happens with probability $1 - p$), then the voters receive the payoff of γ_β if and only if at least one of the K voters who were supposed to vote for B in the default sequence receives a signal, which occurs with probability $1 - (1 - \varepsilon)^K$. the case of $\frac{1-p}{p}\frac{(1-\varepsilon_b)^K}{(1-\varepsilon_a)^L}\frac{\gamma_\beta}{\gamma_\alpha} \geq 1$ is symmetric.

Paying special attention to the voting rule provides a further interesting insight.

Corollary 3. *Consider the general voting model with N voters such that $p = \frac{1}{2}$, $\varepsilon_a = \varepsilon_b$ and $\gamma_\alpha = \gamma_\beta$. The voters' payoff under pure consistent strategy equilibria is strictly increasing in $\max\{K, L\}$. Therefore, it is maximized when $K = N$ or $L = N$, and minimized when $|K - L| \leq 1$.*

This corollary shows that, in terms of the equilibrium payoffs, the majority voting

rule performs the worst while the unanimous rule is the best in an otherwise symmetric setting.

In the Online Appendix, we further solve for the welfare bounds of the inconsistent strategy equilibria in the general asymmetric model.

6.2   Comparison with Static Voting

In this section, we compare our model of sequential voting with its simultaneous-move version. We first consider the upper bound on the payoff in the simultaneous voting game and discuss the strategy profile that achieves the bound. Then we consider a prominent Bayesian Nash equilibrium, the "random voting strategy," and discuss its welfare property.

Formally, in the simultaneous voting game, everything is the same as in our main model except that voters vote simultaneously. This model is characterized by parameters (n, ε). Voter i's strategy is a mapping $\pi_i : \{a, b, \emptyset\} \to \Delta(\{A, B\})$. Let Π_i be the set of all strategies of player i, and $\Pi = \times_{i=1}^{2n+1} \Pi_i$. Bayesian Nash equilibrium of this game can be defined in a standard manner.

Let Π^C be the set of consistent strategy profiles, where consistency is defined analogously to the one in the main model. Let Π^{PC} be the set of pure consistent strategy profiles. Let Π^{n+1} be the set of pure consistent strategy profiles in which there is an action $X \in \{A, B\}$ and a subset of players N' with $n + 1$ voters such that $\pi_i(\emptyset)(X) = 1$ for all $i \in N'$ and $\pi_i(\emptyset)(X) = 0$ for all $i \notin N'$. Note that $\Pi^{n+1} \subset \Pi^{PC} \subset \Pi^C \subset \Pi$.

Notice that for any given strategy profile $\pi \in \Pi$, every voter's ex ante expected payoff is the same. We call this "the expected payoff from π."

Proposition 4. *Consider a simultaneous-move voting game with (n, ε). Then, the following hold.*

1. *There exists $\bar{\varepsilon} > 0$ such that if $\varepsilon \in (0, \bar{\varepsilon})$, then any strategy profile in Π^{n+1} maximizes the expected payoff among Π.*

2. *Consider $\pi \in \Pi^{PC}$. Then, π is a Bayesian Nash equilibrium if and only if $\pi \in \Pi^{n+1}$.*

An implication of this result is that a pure consistent strategy profile in the simultaneous voting game constitutes an equilibrium of our sequential voting model.

This result may superficially look similar to that of Dekel and Piccione (2000), but their result does not imply ours. This is because Dekel and Piccione (2000) has the "full support" assumption that eliminates the possibility of off-path information sets. In our consistent equilibria, however, specifying off-path beliefs and strategies is a crucial step in showing that voters are indeed taking a best response.

Now we turn our attention to one of the most prominent strategy profiles: Call the consistent strategy in which the voter assigns probability $\frac{1}{2}$ to each action regardless of the history of actions the *random voting strategy.*

Proposition 5. *For any simultaneous-move voting game with (n, ε), the strategy profile in which every voter uses the random voting strategy is a Bayesian Nash equilibrium.*

The intuition is simple. Under the random voting strategy profile, a given voter is pivotal only when n other voters vote for A and the remaining n voters vote for B. Under such an event, the posterior of the voter, if she does not receive a signal, assigns probability $\frac{1}{2}$ to each state. Hence, the voter is indifferent between A and B, and thus randomizing is indeed optimal. It is straightforward that the voter receiving a signal finds it optimal to vote for the option that is consistent with her signal.

Now, we compare the welfare from pure consistent equilibria with that of the random voting strategy. Given ε, let the payoff from a pure consistent strategy profile be $C(\varepsilon)$ and the payoff from the random voting strategy profile in the static model be $RV(\varepsilon)$.

Proposition 6. *For any $n \geq 1$, there exists $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$, $RV(\varepsilon) < C(\varepsilon)$.*

That is, the expected payoff from a pure consistent equilibrium is greater than that of the random strategy profile. Although the proof involves some tedious calculus, the intuition is simple. For the sake of argument, imagine that n is large. When $\varepsilon > 0$ is very small, one voter receiving a signal is much more likely than more voters receiving a signal. Hence, consider the case in which exactly one voter receives a signal, say a. In such a case, the event in which this voter would become pivotal under the random voting strategy is small because n is large, and we would need exactly n voters voting for A and exactly n voters voting for B. However, under a pure consistent strategy profile, if this voter happens to be on the default sequence that specifies her to play

B, then she can overturn the outcome because her action, voting for A, affects the subsequent voters' actions. This substantially improves efficiency. In summary, the fact that the voting is sequential makes the probability of a voter becoming pivotal high, and this is the reason why the expected payoff from a pure consistent equilibrium is greater than that of random strategy profile.

6.3   Abstention Model

The main section considered a model without abstention. Such a model would approximate the situation as in *Squid Game* as well as roll call vote in United Nations, US Senate and Japanese Diet where abstention is unavailable. In other applications, however, abstention is allowed. This section discusses a model with abstention.

Formally, there are N voters. The action space is now $\{A, B, \Phi\}$, where Φ stands for abstention. The voting ends with outcome A at voter k if, after k's vote, the number of votes on A minus the number of votes on B is strictly greater than $N - k$. Similarly, the voting ends with outcome B at voter k if, after her vote, the number of votes on B minus the number of votes on A is strictly greater than $N - k$. If the number of votes for A and that for B are equal after N voters vote, then outcomes A and B realize with equal probability.

We say that voter k's strategy is *consistent in the abstention model* if k chooses the "consistent" action if she receives a signal, i.e., $\sigma_k(\cdot, a)(A) = 1$ and $\sigma_k(\cdot, b)(B) = 1$.

In the Online Appendix, we show that there is an equilibrium in which all voters use a strategy that is consistent in the abstention model under any history, and abstain on the path of play if no signal is received. The Online Appendix also provides an example of an equilibrium in which there is a voter who does not use a strategy that is consistent in the abstention model: in particular, she votes for Φ if her signal is a and A if her signal is ϕ (and votes for B if the signal is b). In both examples, the ex ante payoff turns out to be the highest payoff achievable by any social choice function.

Finally, we note that, in the standard voting models in the literature, introduction of abstention would strictly improve the welfare. In our model, however, the highest payoff achievable by any social choice function can be obtained *without* the introduction of abstention as we have seen in Section 4.

6.4 Vote Count Model

One may wonder if our "anything goes" result hinges "too much" on the observation of the particular default sequence. However, the same result can be proved even if we allow voters to observe only the vote counts.

To state this formally, change the base model so that voter k's strategy is a mapping from the numbers of votes for A and k's signals to distributions over her actions: $\sigma_k : \{0, \ldots, k-1\} \times \{a, b, \emptyset\} \to \Delta(\{A, B\})$. Given this definition of strategy, we can define sequential equilibria. Call this model the *vote count model*.

Theorem 7. *In the vote count model, for any $n \geq 0$, $\varepsilon \in (0, 1]$ and any sequence $S \in \mathcal{S}$, there is a pure consistent strategy equilibrium σ in the model with (n, ε) such that S is the default sequence of σ.*

As for Theorem 1, the proof is constructive. The equilibrium construction, however, is different because the voters cannot condition their votes on who deviated first from the default sequence when the observed vote counts contradict the default sequence. For this reason, we consider an alternative construction. There is still a default sequence. If a voter with no signal sees the vote counts that are implied by the default sequence, then she votes according to the default sequence. Otherwise, the voter votes for the option that has received more votes than what the default sequence implies.[21]

7   Conclusion

Sequential voting is used on various occasions, such as in the roll call vote in the United Nations, the US Senate, and the Japanese Diet. In those settings, voters would have information of different degrees of precision to each other. This paper considered common-value sequential voting with informed and uninformed voters: In particular, the uninformed voters learn from previous votes and make voting decisions while considering the impact on subsequent votes. We identified two effects, the posterior effect and the delegation effect, and the balance of these two effects resulted in the "anything goes" result. The welfare is invariant as long as we consider the class of pure consistent equilibria where consistency means that the voter follows her

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[21]We note that this strategy profile is an equilibrium in the model in which the entire history of past votes is observable as well.

signal if she is informed. There are inconsistent equilibria as well, and the welfare from the inconsistent equilibria varies quite a bit. Two modifications of the model are discussed, where the modified models imply long voting because the delegation effect is stronger than the posterior effect for the pivotal (deciding) voter due to the modification.

We end the paper by re-stressing that our model is stylized. A stylized model lets us capture a general insight that underlies various settings, and we discussed some extensions (such as asymmetries, abstention or the vote count model). As we discussed, however, there are still features of real sequential voting that we do not capture in this paper. We view such discrepancies as a wonderful possibility for future work. In such future work, the posterior effect and the delegation effect would be essential building blocks in elucidating the incentives of the voters, as they were in this paper.

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A APPENDIX

A.1 Proof of Theorem 1 and Remark 1

Proof. Fix $k \in \{n+1, n+2, \ldots, 2n+1\}$ and take a sequence of actions $S^* = (X_1, \ldots, X_k) \in \{A, B\}^k$ such that X_k appears exactly $n+1$ times. We consider the case when $X_k = A$. The case for $X_k = B$ is symmetric.

Consider the consistent strategy profile σ satisfying the following conditions:

1. For any $i = 1, \ldots, k$, take any sequence $S = (X_1, \ldots, X_{i-1}) \in \{A, B\}^{i-1}$. That is, S coincides with S^* before voter i. Then, i chooses the action specified in S^* given no signal, i.e.,

$$\sigma_i(X_1, \ldots, X_{i-1}, \emptyset)(X_i) = 1.$$

2. For any $i = 1, \ldots, 2n+1$ and $i' = 1, \ldots, \min\{k, i-1\}$, take any sequence $S = (X_1, \ldots, X_{i'-1}, X'_{i'}, \ldots, X'_{i-1}) \in \{A, B\}^{i-1}$ such that $X'_{i'} \neq X_{i'}$. That is, i' is the first voter who does not follow the sequence $(X_1, \ldots, X_k)$. Then, i chooses the same action as $X'_{i'}$ given no signal, i.e.,

$$\sigma_i(S, \emptyset)(X'_{i'}) = 1.$$

Now we check incentives. First, voters who have received a signal take a best response given condition 2 above.

Second, under any histories described in condition 2, by letting i have a belief that the state is α if $X'_{i'} = A$ and β if $X'_{i'} = B$ (such a belief can easily be shown to be consistent in the sense of Kreps and Wilson (1982)), it is straightforward to see that i is taking a best response.

Third, under any histories described in condition 1, suppose that there has been m times of A's before i. Voter i's posterior on α is then

$$P(\alpha) = \frac{(1 - \varepsilon)^{i-m}}{(1 - \varepsilon)^{i-m} + (1 - \varepsilon)^{m+1}},$$

and her posterior on β is

$$P(\beta) = 1 - P(\alpha) = \frac{(1-\varepsilon)^{m+1}}{(1-\varepsilon)^{i-m} + (1-\varepsilon)^{m+1}}.$$

We consider the following two (exhaustive) cases.

1. Suppose that $X_i = A$.

   If i plays A, her payoff is:

   $$P(\alpha) + (1 - P(\alpha))(1 - (1-\varepsilon)^{n-m}).$$

   Suppose now that i plays B. If the state is β, then outcome B realizes with probability 1 given that the voters follow σ. If the state is α, the outcome becomes A if and only if at least $n + 1 - m$ subsequent voters receive signal a. Thus, her payoff is

   $$1 - P(\alpha) + P(\alpha) \cdot Y,$$

   where $Y := \mathrm{Prob}(n + 1 - m \text{ or more signals out of } 2n + 1 - i)$.

   Thus, the payoff from playing A is no less than the payoff from playing B if and only if:

   $$P(\alpha) + (1 - P(\alpha))(1 - (1-\varepsilon)^{n-m}) \geq 1 - P(\alpha) + P(\alpha)Y \iff$$

   $$P(\alpha)[1 + (1-\varepsilon)^{n-m} - Y] \geq (1-\varepsilon)^{n-m} \iff P(\alpha) \geq \frac{(1-\varepsilon)^{n-m}}{(1-\varepsilon)^{n-m} + 1 - Y}.$$

   Now, notice that

   $$\begin{aligned} P(\alpha) &= \frac{(1-\varepsilon)^{i-m}}{(1-\varepsilon)^{i-m} + (1-\varepsilon)^{m+1}} \\ &= \frac{(1-\varepsilon)^{n-m}}{(1-\varepsilon)^{n-m} + (1-\varepsilon)^{n+1+m-i}}. \end{aligned}$$

   Hence, the payoff from playing A is no less than the payoff from playing B if and only if:

   $$(1-\varepsilon)^{n+1+m-i} \leq 1 - Y,$$

or

$$(1 - \varepsilon)^{n+1+m-i} \leq \mathrm{Prob}(n - m \text{ or less signals out of } 2n + 1 - i).$$

Now, let $Z_1 := (n + 1) + (m - i)$ and $Z_2 := (n + 1) + (n - i)$. Notice that $Z_1 \leq Z_2$ because $m \leq n$. The payoff from playing A is no less than the payoff from playing B if and only if:

$$(1 - \varepsilon)^{Z_1} \leq \mathrm{Prob}(Z_2 - Z_1 \text{ or less signals out of } Z_2). \tag{1}$$

Note that

$$\mathrm{Prob}(Z_2 - Z_1 \text{ or less signals out of } Z_2)$$

$$\geq \mathrm{Prob}(\text{The last } Z_1 \text{ voters do not receive a signal}) \tag{2}$$

$$= (1 - \varepsilon)^{Z_1}.$$

Thus, eq. (1) indeed holds, and therefore, the payoff from playing A is no less than the payoff from playing B.

2. Suppose that $X_i = B$.

First, consider the case when i plays B.

Suppose the state is α, which happens with probability $P(\alpha)$. In this case, i's playing B ensures that there is probability 1 that A will be chosen as the outcome.

Suppose the state is β, which happens with probability $1 - P(\alpha)$. Since there have been m times of A right after i plays B and there are $n + 1$ times of A in the given sequence, there will be $n + 1 - m$ times of A at which the voter, upon receiving signal b, can change the outcome to B. The probability that at least one voter out of $n + 1 - m$ voters receives signal b is $1 - (1 - \varepsilon)^{n+1-m}$.

Thus, overall, the payoff from i's voting for B is

$$P(\alpha) + (1 - P(\alpha))(1 - (1 - \varepsilon)^{n+1-m}).$$

Second, consider the case when i plays A.

If the state is α, then outcome A realizes with probability 1 given that the voters follow σ. If the state is β, the outcome becomes B if and only if at least

$(n + 1) - (i - (m + 1))$ subsequent voters receive signal b. Thus, her payoff is

$$P(\alpha) + (1 - P(\alpha))\hat{Y},$$

where $\hat{Y} := \text{Prob}((n + 1) - (i - (m + 1)))$ or more "b" signals out of $2n + 1 - i)$. Thus, the payoff from playing B is no less than the payoff from A if and only if:

$$P(\alpha) + (1 - P(\alpha))(1 - (1 - \varepsilon)^{n+1-m}) \geq P(\alpha) + (1 - P(\alpha))\hat{Y} \iff$$

$$(1 - \varepsilon)^{n+1-m} \leq 1 - \hat{Y},$$

or

$$(1 - \varepsilon)^{n+1-m} \leq \text{Prob}(n + 1 + m - i \text{ or less "}b\text{" signals out of } 2n + 1 - i).$$

Now, let $\hat{Z}_1 := n + 1 - m$ and $\hat{Z}_2 := 2n + 2 - i$. Notice that $\hat{Z}_1 \leq \hat{Z}_2$ because $\hat{Z}_2 - \hat{Z}_1 = 2n + 2 - i - (n + 1 - m) = n + 1 - (i - m) \geq 0$. The payoff from playing B is no less than the payoff from A if and only if:

$$(1 - \varepsilon)^{\hat{Z}_1} \leq \text{Prob}(\hat{Z}_2 - \hat{Z}_1 \text{ or less "}b\text{" signals out of } \hat{Z}_2 - 1). \qquad (3)$$

Note that

$$\begin{aligned}
&\text{Prob}(\hat{Z}_2 - \hat{Z}_1 \text{ or less "}b\text{" signals out of } \hat{Z}_2 - 1) \\
&> \text{Prob}(\hat{Z}_2 - \hat{Z}_1 \text{ or less "}b\text{" signals out of } \hat{Z}_2) \qquad (4) \\
&\geq \text{Prob}(\text{The last } \hat{Z}_1 \text{ voters do not receive an "}b\text{" signal}) \\
&= (1 - \varepsilon)^{\hat{Z}_1}.
\end{aligned}$$

Thus, eq. (3) indeed holds, and therefore, the payoff from playing B is no less than the payoff from A.

Overall, playing X_i is a best response for voter i. $\qquad \square$

Proof of Remark 1. The voters on the default sequence is indifferent if and only if

eq. (2) holds with equality in Case 1 (i.e., $X_k = A$).[22] This happens when either $Z_1 = Z_2$ or $Z_1 = 0$. Note that $Z_1 = Z_2$ is equivalent to $m = n$, and $Z_1 = 0$ is equivalent to $i - 1 = n + m$, so i has seen n times of B. Since eq. (2) is in the case when $X_i = X_k = A$, these are exactly the cases when voter i is deciding. □

A.2 Proof of Theorem 3

Proof. For any number of voters $2n + 1$, the following strategy profile $\bar{\sigma}$ is an equilibrium and achieves the ex ante payoff $\bar{R}$.

- For any $k = 1, \ldots, n$, voter k chooses B if she receives a; otherwise she plays A.

- For any $k = n + 1, \ldots, 2n + 1$, voter k uses a consistent strategy. If he does not receive a signal, then (i) k plays B if he observes that voters $1, \ldots, n$ play A and voters $n + 1, \ldots, k - 1$ play B, and (ii) k plays A otherwise.

Note that this is a pure strategy profile and is not consistent. Its default sequence is $(A, \ldots, A, B, \ldots, B)$, where A continues n times and then B continues $n + 1$ times. To see that each voter chooses a best response, observe first that it is immediate that any voter who receives a signal is taking a best response. Also, observe that once there is a deviation from the default sequence, then voters take a best response given a belief that the first deviator from the default sequence has received a signal and followed the equilibrium strategy (such a belief can easily be shown to be consistent in the sense of Kreps and Wilson (1982)). So suppose that the voters 1 through $k - 1$ have followed the default sequence, and suppose that voter k receives no signal. The posterior on α is $P(\alpha) = \frac{(1-\varepsilon)^k}{(1-\varepsilon)^k + (1-\varepsilon)}$. If k follows the specified strategy, then her payoff is

$$P(\alpha)(1 - (1 - \varepsilon)^{2n+1-k}) + (1 - P(\alpha)) \cdot 1. \tag{5}$$

If k deviates, her payoff is $P(\alpha)$ if $k \geq n + 1$. In this case, k is taking a best response because (5) is larger than $P(\alpha)$ due to $P(\alpha) < \frac{1}{2} < 1 - P(\alpha)$.

If $k \leq n$, then her payoff from the deviation is

$$P(\alpha) + (1 - P(\alpha))\tilde{Y},$$

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[22]Note that indifference does not occur in Case 2 because of the strict inequality in eq. (4) of the proof above.

where $\tilde{Y} := \mathrm{Prob}(n$ or more signals out of $2n - k)$.

Thus, the payoff from playing A is no less than the payoff from playing B if and only if

$$P(\alpha)(1 - (1 - \varepsilon)^{2n+1-k}) + (1 - P(\alpha)) \cdot 1 \geq P(\alpha) + (1 - P(\alpha))\tilde{Y},$$

or

$$P(\alpha) \leq \frac{1 - \tilde{Y}}{1 - \tilde{Y} + (1 - \varepsilon)^{2n+1-k}}.$$

Since $P(\alpha) = \frac{(1-\varepsilon)^k}{(1-\varepsilon)^k + (1-\varepsilon)}$, this is equivalent to:[23]

$$\frac{(1 - \varepsilon)^k}{1 - \varepsilon} \leq \frac{1 - \tilde{Y}}{(1 - \varepsilon)^{2n+1-k}},$$

or

$$(1 - \varepsilon)^{2n} \leq 1 - \tilde{Y}. \tag{6}$$

Now, note that $\tilde{Y}$ is equal to Y defined in the proof of Theorem 1 where we set $m = 1$ and $i = k + 1$. Since that proof shows $(1 - \varepsilon)^{n+1+m-i} \leq 1 - Y$, we have

$$(1 - \varepsilon)^{n+1+1-(k+1)} \leq 1 - \tilde{Y},$$

or

$$(1 - \varepsilon)^{n+1-k} \leq 1 - \tilde{Y}.$$

Since $k, n \geq 1$, we have $(1 - \varepsilon)^{2n} \leq (1 - \varepsilon)^n \leq (1 - \varepsilon)^{n+1-k}$, which implies (6). This shows that k is taking a best response when $k \leq n$.

Finally, since the payoff is 0 under $\bar{\sigma}$ if and only if the state is α and no one receives a signal, the expected payoff is $1 - \frac{(1-\varepsilon)^{2n+1}}{2}$. $\qquad\square$

A.3   Proof of Theorem 4

Proof. Consider the following strategy profile, which we denote by $\underline{\sigma}$.

- Voter $k = 1, \ldots, n$: If all the actions observed so far are A, then play A irrespective of the signal. Otherwise, play a consistent strategy in which A is played if no signal is received.

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[23]Here we use the fact that, for any $a, b, c, d > 0$, $\frac{a}{a+b} \leq \frac{c}{c+d} \iff \frac{1}{1+(b/a)} \leq \frac{1}{1+(d/c)} \iff \frac{a}{b} \leq \frac{c}{d}$.

- Voter $k = n + 1, \ldots, 2n$: If all the actions observed so far are n times of A followed by $k-1-n$ times of B, then play B irrespective of the signal. Otherwise, play a consistent strategy in which A is played if no signal is received.

- Voters $k = 2n + 1$: Play a consistent strategy in which A is played if no signal is received.

Note that this strategy profile has the ex ante payoff of $\frac{1}{2} + \frac{1}{2}\varepsilon = \frac{1+\varepsilon}{2}$. We show that, if $n \geq 2$, $\underline{\sigma}$ constitutes an equilibrium with a belief that, after any deviation, any voter who has not received a signal assigns probability 1 to α (such a belief can easily be shown to be consistent in the sense of Kreps and Wilson (1982)).

Consider voter k. Fix any history of actions by the previous voters. First, suppose that k receives signal a. Then, since the outcome will be A if k follows $\underline{\sigma}_k$, playing $\underline{\sigma}_k$ induces the expected payoff of 1, which is the highest possible payoff in this game. Hence, $\underline{\sigma}_k$ is a best response.

Second, suppose that k receives signal b.

- Suppose that the voters so far have followed $\underline{\sigma}$.

    - If $1 \leq k \leq n$, then if she follows $\underline{\sigma}_k$ then her payoff is ε. If instead she plays B then at least n voters from the set of subsequent voters have to receive signal b in order for k to expect the payoff of 1, and otherwise she receives the payoff of 0. Hence, her payoff is $O(\varepsilon^n)$. Since $n \geq 2$, this implies that there exists $\bar{\varepsilon} > 0$ such that for all $\varepsilon < \bar{\varepsilon}$, following $\underline{\sigma}_k$ is a best response.

    - If $n + 1 \leq k \leq 2n$, then playing A ends the game with outcome A, so it induces the expected payoff of 0. Hence, playing B is a best response.

    - If $k = 2n + 1$, then playing B induces the payoff of 1 with probability 1, so it is a best response to play B.

- Suppose that there is a voter who has deviated from $\underline{\sigma}$. Then, the play by the subsequent voters will not be affected by the action taken by voter k. Hence, it is a best response for voter k to provide an additional vote for B. Therefore, playing B is a best response.

Third, suppose that voter k did not receive a signal.

- Suppose that the voters so far have followed $\underline{\sigma}$. Then, the posterior belief on α is $\frac{1}{2}$.

– Suppose $k \leq n$. If k follows $\underline{\sigma}_k$, then her payoff is $\frac{1}{2} + \frac{1}{2}\varepsilon$. If she does not follow $\underline{\sigma}_k$, then her payoff is $\frac{1}{2} + \frac{1}{2}O(\varepsilon^n)$. Since $n \geq 2$, following $\underline{\sigma}_k$ is a best response.

– Suppose $n + 1 \leq k \leq 2n$. If k follows $\underline{\sigma}_k$, then her payoff is $\frac{1}{2} + \frac{1}{2}\varepsilon$. If she does not follow $\underline{\sigma}_k$, then her payoff is $\frac{1}{2}$. Hence, following $\underline{\sigma}_k$ is a best response.

– Suppose $k = 2n + 1$. Then, k is indifferent between the two actions. So following $\underline{\sigma}_k$ is a best response.

• Suppose that there is a voter who has deviated from $\underline{\sigma}$ before k's move. Then, the play by the subsequent voters will not be affected by the action taken by voter k. Hence it is a best response for k to provide an additional vote for A because k's belief assigns probability 1 to α.

This completes the proof. □

A.4   Proof of Proposition 2

Proof. Fix a pure strategy equilibrium σ. Take the default sequence of σ. Suppose without loss of generality that the last voter on this default sequence, whom we denote voter k^*, plays A.

First, we show the following lemma.

Lemma 1. *Suppose that the votes by voters $1, \ldots, k^* - 1$ have followed the default sequence of σ. Then, voter k^* votes for B if and only if she receives signal b.*

Proof of Lemma 1. Consider the history in which voters $1, \ldots, k^* - 1$ have followed the default sequence of σ. Given the assumption that k^* plays A given no signal, it suffices to check the cases when she receives a signal.

1. First, we show that k^* plays B if she receives signal b. We show this using induction on k. To do this, fix $k' \in \{k^*, \ldots, 2n + 1\}$. Suppose, as an induction hypothesis, that for every $k > k'$, voter k plays B if he receives signal b.[24] Suppose that voter k' receives signal b. Given this signal, her posterior probability on β is 1. If k' votes for A, then her expected payoff is 0. If k' votes for B,

___
[24]Note that this hypothesis is vacuously true when $k' = 2n + 1$.

then there is a strictly positive probability that all the subsequent voters receive signal b and, in that case, the game ends with outcome B by the induction hypothesis. Hence, her expected payoff is strictly positive. Thus, playing B is a unique best response for k', and hence, voter k' plays B if she receives signal b. This shows that voter k^* plays B if she receives signal b.

2. Next, we show that k^* plays A if she receives signal a. To see this, suppose to the contrary that k^* plays B if she receives signal a. In this case, the outcome must be A with probability 1 after k^* plays B because otherwise, playing B would give k^* a payoff strictly less than 1, while she would get the payoff of 1 if she played A, making her choice B suboptimal. This implies that the outcome will be A if k^* chooses B when the state is α, no matter what her signal is. Now, suppose that k^* did not receive a signal. Let $P(\alpha)$ be the posterior of k^* at such an information set. Then, the expected payoff of voter k^* is $P(\alpha)$ if she plays A. If she instead plays B, then the outcome will be A with probability 1 if the state is α as we have concluded. The argument in item 1 above implies that if the state is β and all the subsequent voters receive signal b, which happens with probability $\varepsilon^{2n+1-k^*} > 0$, then the outcome is B. Hence, the expected payoff of voter k^* is at least $P(\alpha) + (1 - P(\alpha))\varepsilon^{2n+1-k'}$ if she plays B, and this is strictly greater than $P(\alpha)$ because $1 - P(\alpha) > 0$ and $\varepsilon > 0$. Hence, playing A is suboptimal for k^* when she does not receive a signal, which contradicts the assumption that she votes for A given no signal. Thus, k^* plays A if she receives signal a.

$\square$

Given Lemma 1, once k^* plays B, the subsequent voters assign posterior probability 1 to state β, so the outcome will be B with probability 1.

Lemma 2. *Under σ, if any voter receives signal a when the votes so far followed the default sequence, then she expects that the outcome will be A with probability 1.*

Proof of Lemma 2. We use induction. Fix $k \leq k^*$. Suppose as an induction hypothesis that for every $k' \in \{k + 1, \ldots, k^*\}$, if the actions by voters $1, \ldots, k' - 1$ have followed the default sequence and k' receives signal a, then the outcome will be A with probability 1. Then, suppose that the actions by voters $1, \ldots, k-1$ have followed the default sequence and k receives signal a. If k plays the action specified in the

default sequence, then either (i) no subsequent voters receive a signal, or (ii) there is at least one subsequent voter who receives signal a. In case (i), by the definition of the default sequence, the outcome will be A with probability 1. In case (ii), the first voter who receives signal a expects that the outcome will be A, which follows from the induction hypothesis. Hence, the outcome will be A in either case. Therefore, k's expected payoff is 1 if k plays the action specified in the default sequence. Since k under σ must be doing at least as good as playing any action, this implies that, under σ, k expects the payoff of 1, and hence she expects that the outcome will be A with probability 1. This completes the induction argument and completes the proof. □

Lemma 3. *Under σ, if any voter receives signal b when the votes so far followed the default sequence, then she expects that the outcome will be B with probability at least ε.*

Proof of Lemma 3. We use induction. Fix $k \leq k^* - 1$. Suppose as an induction hypothesis that for every $k' \in \{k+1, \ldots, k^*-1\}$, if the actions by voters $1, \ldots, k'-1$ have followed the default sequence and k' receives signal b, then the outcome will be B with probability at least ε. Then, suppose that the actions by voters $1, \ldots, k-1$ have followed the default sequence and k receives signal b. If k plays the action specified in the default sequence, then either (i) no subsequent voters in $\{k+1, \ldots, k^*-1\}$ receive a signal, or (ii) there is at least one subsequent voter in $\{k+1, \ldots, k^*-1\}$ who receives signal b. In case (i), by Lemma 1, voter k^* will play B if she receives signal b, and thus there is probability ε that the outcome will be B as we concluded after Lemma 1. In case (ii), the first voter who receives signal b expects that the outcome will be B with a probability of at least ε, which follows from the induction hypothesis. Hence, the outcome will be B with at least ε probability in either case. Therefore, k's expected payoff under σ must be at least ε if she plays the action specified in the default sequence. Since k under σ must be doing at least as good as playing any action, this implies that, under σ, k expects a payoff of at least ε, and hence she expects that the outcome will be B with probability at least ε. This completes the induction argument. Therefore, we have shown that, if any voter receives signal b, then she expects that the outcome will be B with probability at least ε. □

Finally, consider voter 1. If the state is α, then either (i) no voters in $\{1, \ldots, k^*\}$ receive a signal, or (ii) there is at least one voter in $\{1, \ldots, k^*\}$ who receives signal a. In case (i), the outcome will be A with probability 1 by the choice of the default

39

sequence. In case (ii), take the first voter who has received signal a. Lemma 2 shows that this voter expects the outcome to be A with probability 1. Hence, if the state is α, voter 1's expected payoff is 1.

If the state is β, then either (i) no voters in $\{1, \ldots, k^* - 1\}$ receive a signal, or (ii) there is at least one voter in $\{1, \ldots, k^* - 1\}$ who receives signal b. In case (i), voter k^* will have seen the history that has followed the default sequence. If she receives no signal, which happens with probability $1 - \varepsilon$, then she plays A by assumption, and thus the outcome will be A with probability 1. If she receives signal b, which happens with probability ε, then the outcome will be B with probability 1 as we concluded after Lemma 1. Overall, voter 1 expects the probability that the outcome will be B to be ε conditional on (i). In case (ii), take the first voter who has received signal b. Lemma 3 shows that this voter assigns a probability of at least ε that the outcome will be B. Thus, conditional on the event that (ii) realizes, voter 1 expects that the probability that the outcome will be B is at least ε. Therefore, if the state is β, the outcome will be B with at least ε probability.

Overall, since the prior on the states α and β are both $\frac{1}{2}$, voter 1's expected payoff is at least

$$\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \varepsilon = \frac{1 + \varepsilon}{2}.$$

$\square$

A.5   Proof of Theorem 5

We first introduce a model that generalizes the one in Section 5.1.

Consider a game in which a single voter k^* is constrained to choose a mixed action. Formally, for any $\xi \in (0, \frac{1}{2}]$, a (k^*, ξ)-*tremble game* is a game that is the same as the one in the main section, except that voter k^*, upon moving, chooses a probability distribution over $\{A, B\}$ such that each action receives probability at least ξ if he does not receive a signal. We say that a strategy profile in a (k^*, ξ)-tremble game is *pseudo-pure* if every voter except k^* uses a pure strategy.

Theorem 5'. *In any (k^*, ξ)-tremble game, there is no consistent pseudo-pure strategy equilibrium such that the game ends before reaching voter k^* when no voter receives a signal.*

Note that Theorem 5' implies Theorem 5 by setting $k^* = 2n + 1$.

Proof of Theorem 5'. Fix a pseudo-pure consistent strategy equilibrium. Suppose that if no voter gets a signal, on the path of play, the game ends before reaching voter k^*. Let k be the voter who ends the game in this case. Suppose without loss of generality that k plays A on the default sequence.

Voter k has seen n times of action A and $k-n-1$ times of action B. The posterior on α is thus

$$P(\alpha) = \frac{(1-\varepsilon)^{k-n}}{(1-\varepsilon)^{k-n} + (1-\varepsilon)^{n+1}} = \frac{1}{1 + (1-\varepsilon)^{2n+1-k}}. \tag{7}$$

If voter k plays A, then the game ends and her expected payoff becomes $P(\alpha)$.

If voter k votes for B instead, then by Bayes rule each subsequent voter who receives her turn to vote and receives no signal assigns probability 1 to state β. This implies that she votes for B because voting for A results in the expected payoff of 0 while voting for B ensures a strictly positive payoff due to the event in which all the subsequent voters receive signal b.

Let q be the probability that voter k^*'s strategy assigns to A when he does not receive a signal. Then, k's payoff if she plays B is

$$P(\alpha) \left[1 - (1-q)(1-\varepsilon)^{2n+1-k}\right] + (1 - P(\alpha)) \left[\varepsilon + (1-\varepsilon)(1-q)\right].$$

Let this be $f(q)$. Note that

$$f(0) = P(\alpha)[1 - (1-\varepsilon)^{2n+1-k}] + (1 - P(\alpha)) = 1 - P(\alpha)(1-\varepsilon)^{2n+1-k} = P(\alpha).$$

Then, for $q > 0$,

$$\frac{f(q) - P(\alpha)}{q} = \frac{f(q) - f(0)}{q}$$
$$= P(\alpha)(1-\varepsilon)^{2n+1-k} - (1 - P(\alpha))(1 - \varepsilon)$$
$$= \frac{(1-\varepsilon)^{(2n+1-k)}}{1 + (1-\varepsilon)^{2n+1-k}} - \frac{(1-\varepsilon)^{(2n+1-k)}}{1 + (1-\varepsilon)^{2n+1-k}}(1 - \varepsilon)$$
$$= \frac{\varepsilon(1-\varepsilon)^{(2n+1-k)}}{1 + (1-\varepsilon)^{2n+1-k}} > 0.$$

So, $f(q) > P(\alpha)$ for any $q > 0$. Since our assumption implies that $q \geq \xi > 0$, this means that k would be better off playing B than A. This is a contradiction to the assumption that k plays A in the fixed equilibrium, which completes the proof. $\square$

A.6 Proof of Theorem 6

Proof. Fix a pure consistent equilibrium in a pivot-aversion game and suppose, towards contradiction, that its default sequence has length $l \leq 2n$. Without loss of generality, let A be the action that l takes when she has observed actions specified in the default sequence and does not receive a signal. Her posterior belief about the state α in this case is

$$P(\alpha) = \frac{(1-\varepsilon)^{l-n}}{(1-\varepsilon)^{l-n} + (1-\varepsilon)^{n+1}}.$$

By choosing A, her payoff is $P(\alpha) - c$. If she chooses B, then all subsequent voters have a belief that assigns probability 1 to state β unless they receive signal a. Since voting for A ends the voting with outcome A, this implies that all subsequent voters vote for A if and only if they receive signal a. This implies that l's expected payoff from choosing B is

$$P(\alpha)(1 - (1-\varepsilon)^{2n+1-l}) + (1 - P(\alpha))$$

(note that we do not subtract c because there have been $l - 1 - n$ times of B, and $l - 1 - n \leq 2n - 1 - n < n$), which is equal to

$$\frac{(1-\varepsilon)^{l-n}(1 - (1-\varepsilon)^{2n+1-l}) + (1-\varepsilon)^{n+1}}{(1-\varepsilon)^{l-n} + (1-\varepsilon)^{n+1}} = \frac{(1-\varepsilon)^{l-n}}{(1-\varepsilon)^{l-n} + (1-\varepsilon)^{n+1}} = P(\alpha).$$

Since $c > 0$, choosing B is strictly better than choosing A, which implies that voter l is not taking a best response. Contradiction. $\square$

A.7 Proof of Proposition 3

The proof is an adaptation of the proofs of other results that we have already presented. In particular, to show that any sequence that leads to a particular outcome can be a default sequence of a pure consistent equilibrium, we directly generalize the proof of Theorem 1. To show that no sequence that leads to a particular outcome can be a default sequence of a pure consistent equilibrium, we closely follow the proof of Theorem 5. For this reason, we omit the complete proof of Proposition 3 here but

provide its details in the Online Appendix.

A.8 Proof of Proposition 4

Part 1:

The proof is conducted in three steps. In the first step, we show that for any general strategy profile, one can find a consistent strategy profile that yields a weakly greater expected payoff. The second step shows that for any consistent strategy profile, there is a pure consistent strategy profile that yields a weakly greater expected payoff. In the third step, we show that any strategy profile in Π^{n+1} maximizes the expected payoff among the set of pure consistent strategy profiles when $\varepsilon > 0$ is small, which proves the desired result.

Step 1: Take any strategy profile $\pi \in \Pi$ and fix any voter i. Let π' be a strategy profile such that $\pi'_j(s) = \pi_j(s)$ for every $j \in I$ and $s \in \{a, b, \emptyset\}$ except that $\pi'_i(a)(A) = 1$.

We show that the expected payoff from π' is weakly greater than the expected payoff from π. To see this, notice that there are four possible realizations of the pair of the state and i's signal: (α, a), $(\alpha, \emptyset)$, (β, b), and $(\beta, \emptyset)$. Of these four, the distribution over the action profiles under π' is identical to the one under π given the last three pairs. When the pair is (α, a), the distribution over the number of votes on A under π' is identical to or first-order stochastically dominates that of the number of votes on A under π. Hence, the expected payoff from π' is weakly greater than the expected payoff from π.

Iteratively using this argument for signals a and b for all voters shows that, for any $\pi \in \Pi$, one can find $\hat{\pi} \in \Pi^C$ such that the expected payoff from $\hat{\pi}$ is weakly greater than the expected payoff from π.

Step 2: Take any $\pi \in \Pi^C$ and fix any voter i. Let π' and π'' be two strategy profiles such that $\pi'_j(s) = \pi''_j(s) = \pi_j(s)$ for every $j \in I$ and $s \in \{a, b, \emptyset\}$ except that $\pi'_i(\emptyset)(A) = 1$ and $\pi''_i(\emptyset)(B) = 1$. Since the expected payoff is linear in i's mixing probability when she does not receive a signal, the higher of the expected payoffs from π' and π'' is no less than the expected payoff from π.

Iteratively using this argument for all voters shows that, for any $\pi \in \Pi^C$, one can find $\hat{\pi} \in \Pi^{PC}$ such that the expected payoff from $\hat{\pi}$ is weakly greater than the expected payoff from π.

Step 3: Take any $\pi, \pi' \in \Pi^{PC}$. Let I_A be the set of voters such that $i \in I_A$ if and only if $\pi_i(\emptyset)(A) = 1$. Let I_B be the set of voters not in I_A. Similarly, let I'_A be the set of voters such that $i \in I'_A$ if and only if $\pi'_i(\emptyset)(A) = 1$. Let I'_B be the set of voters not in I'_A. Suppose that $|I'_A| > |I_A| > |I_B| > |I'_B|$. Then, under both π and π', if the state is α, then the outcome will be A with probability 1, and hence the expected payoff is 1 in this case. If the state is β, then the outcome is B if and only if $\frac{|I_A| - |I_B| + 1}{2}$ or more voters who are in I_A receive a signal under π, and similarly, it is B if and only if $\frac{|I'_A| - |I'_B| + 1}{2}$ or more voters who are in I'_A receive a signal under π'. We claim that, conditional on state β, the outcome is more likely to be B under π than under π'.

To see this, first, for ease of notation, let $\frac{|I_A| - |I_B| + 1}{2} = k$ and $|I'_A| - |I_A| = l$. Note that $\frac{|I'_A| - |I'_B| + 1}{2} = \frac{(|I_A| + l) - (|I_B| - l) + 1}{2} = \frac{|I_A| - |I_B| + 1}{2} + l = k + l$. Take an arbitrary set I''_A such that $I''_A \subseteq I'_A$ and $|I''_A| = |I_A|$. We have that

Prob(the outcome is $B | \beta, \pi'$)

$= \text{Prob}(k + l \text{ or more ``} b \text{'' signals out of } |I_A| + l)$

$< \text{Prob}(k \text{ or more but less than } k + l \text{ ``} b \text{'' signals are received by voters in } I''_A)$

$\quad + \text{Prob}(k + l \text{ or more ``} b \text{'' signals are received by voters in } I''_A)$

$= \text{Prob}(k \text{ or more but less than } k + l \text{ ``} b \text{'' signals out of } |I_A|)$

$\quad + \text{Prob}(k + l \text{ or more ``} b \text{'' signals out of } |I_A|)$

$= \text{Prob}(k \text{ or more ``} b \text{'' signals out of } |I_A|)$

$= \text{Prob}(\text{the outcome is } B | \beta, \pi).$

This proves our claim and thus, the expected payoff under π is greater than under π'.

Hence, since the symmetric argument can be made when $|I'_A| < |I_A| < |I_B| < |I'_B|$, if $|I_A| = n + 1$ or $|I_B| = n + 1$ holds for a given π, then π maximizes the expected payoff in Π^{PC}. Therefore, by Steps 1 and 2, under the same condition, π maximizes the expected payoff in Π as well.

Part 2:

The "if" direction: By the argument in Step 1 of the proof of Part 1, for any player, playing a consistent strategy is a best response when receiving a signal.

Fix $\pi \in \Pi^{n+1}$. Take the sets I_A and I_B as in Step 3 of the proof of Part 1. Suppose without loss of generality that $|I_A| = n + 1$ and $|I_B| = n$. Consider $i \in I_A$ and

suppose she does not receive a signal. Since $|I_A \setminus \{i\}| = |I_B|$, the action distribution of the opponents is perfectly symmetric for i. Hence, since i's vote does not affect other voters' signals or actions, she is indifferent between the two actions. Thus in particular, voting for A is a best response.

Next, consider $i \in I_B$ and suppose she does not receive a signal. Since $|I_A| = n+1$, the only case in which i's vote affects the outcome is when some voter in I_A has received signal b and votes for B. This happens only when the state is β. Thus, since i's vote does not affect other voters' signals or actions, voting for B is a best response for i.

The "only if" direction: Fix $\pi \in \Pi^{PC} \setminus \Pi^{n+1}$. Take the sets I_A and I_B as in Step 3 of the proof of Part 1. Suppose without loss of generality that $|I_A| > |I_B|$. Consider $i \in I_A$. Since $|I_A \setminus \{i\}| > |I_B|$, the only case in which i's vote affects the outcome is when some voter in I_A has received signal b and votes for B. This happens only when the state is β. Thus, since i's vote does not affect other voters' signals or actions and i can be pivotal (there are exactly n other voters voting for each option) with a strictly positive probability, voting for A is strictly suboptimal for i. Hence, π is not a Bayesian Nash equilibrium.

A.9   Proof of Proposition 5

Proof. Fix the strategy profile in which every voter chooses the random voting strategy, and consider voter k who has not received a signal and is about to choose between A or B. The only case in which k's vote affects k's payoff is when exactly n other voters choose A and n other voters choose B. When this happens, k's posterior on each state is

$$\frac{\frac{1}{2}(\varepsilon + \frac{1}{2}(1-\varepsilon))^n(\frac{1}{2}(1-\varepsilon))^n}{\frac{1}{2}(\varepsilon + \frac{1}{2}(1-\varepsilon))^n(\frac{1}{2}(1-\varepsilon))^n + \frac{1}{2}(\varepsilon + \frac{1}{2}(1-\varepsilon))^n(\frac{1}{2}(1-\varepsilon))^n} = \frac{1}{2}.$$

Hence, k is indifferent between choosing A and choosing B.

If voter k receives a signal, then it is obviously a best response to assign probability 1 to the corresponding action given that her action does not affect the actions by the subsequent voters. $\qquad\square$

A.10   Proof of Proposition 6

Proof. First, we know that the conclusion of the statement is true for $n = 1$. Hence, below we consider the case of $n \geq 2$. We have

$$C(\varepsilon) = \frac{1}{2} \cdot 1 + \frac{1}{2}(1 - (1 - \varepsilon)^{n+1}).$$

On the other hand, we have

$RV(\varepsilon)$

$= \text{Prob(no voter receives a signal)} \cdot \text{Prob}(n + 1 \text{ or more voters who receive no signal vote for } X)$

$+ \text{Prob(1 voter receives a signal)} \cdot \text{Prob}(n \text{ or more voters who receive no signal vote for } X)$

$+ \text{Prob(2 voters receive a signal)} \cdot \text{Prob}(n - 1 \text{ or more voters who receive no signal vote for } X)$

$+ \cdots$

$+ \text{Prob}(n - 1 \text{ voters receive a signal}) \cdot \text{Prob(2 or more voters who receive no signal vote for } X)$

$+ \text{Prob}(n \text{ voters receive a signal}) \cdot \text{Prob(1 or more voters who receive no signal vote for } X)$

$+ \text{Prob}(n + 1 \text{ or more voters receive a signal}) \cdot 1,$

where X is the action corresponding to the realized state. We can show that $RV(\varepsilon)$ has the following upper bound $\overline{RV}(\varepsilon)$:

$$RV(\varepsilon) \leq \overline{RV}(\varepsilon) := \text{Prob(no voters receive a signal)} \cdot \frac{1}{2}$$

$$+ \text{Prob(1 voter receives a signal)} \cdot \frac{1 + g(n)}{2}$$

$$+ \text{Prob(2 or more voters receive a signal)} \cdot 1,$$

where $g(n)$ is the probability that exactly n voters out of $2n$ voters, conditional on not receiving a signal, vote for A, and we have

$$g(n) = \frac{(2n)!}{n!n!} \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^n = \frac{(2n)!}{n!n!} \left(\frac{1}{2}\right)^{2n}.$$

Note also that

$$\text{Prob}(1 \text{ voter receives a signal}) = (2n+1)\varepsilon(1-\varepsilon)^{2n}.$$

Hence,

$$\overline{RV}(\varepsilon) = (1-\varepsilon)^{2n+1}\frac{1}{2} + (2n+1)\varepsilon(1-\varepsilon)^{2n}\frac{1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]}{2} + O(\varepsilon^2).$$

Thus,

$$C(\varepsilon) - \overline{RV}(\varepsilon)$$

$$= \frac{1}{2}(1-(1-\varepsilon)^{n+1}) - \left[-(1-(1-\varepsilon)^{2n+1})\frac{1}{2} + (2n+1)\varepsilon(1-\varepsilon)^{2n}\frac{1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]}{2} + O(\varepsilon^2) \right]$$

$$= 1 - \frac{1}{2}\left[(1-\varepsilon)^{n+1} + (1-\varepsilon)^{2n+1} + (2n+1)\varepsilon(1-\varepsilon)^{2n}\left(1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]\right)\right] - O(\varepsilon^2)$$

Now, let

$$h(\varepsilon) := (1-\varepsilon)^{n+1} + (1-\varepsilon)^{2n+1} + (2n+1)\varepsilon(1-\varepsilon)^{2n}\left(1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]\right).$$

We have

$$h'(\varepsilon) = -(n+1)(1-\varepsilon)^n - (2n+1)(1-\varepsilon)^{2n}$$
$$+ (2n+1)\left(1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]\right)\left((1-\varepsilon)^{2n} - 2n\varepsilon(1-\varepsilon)^{2n-1}\right).$$

Hence,

$$h'(0) = -(n+1) - (2n+1) + (2n+1)\left(1 + \left[\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right]\right)$$

$$= -(n+1) + (2n+1)\left(\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n}\right).$$

Now, note that $\frac{(2n)!}{n!n!}\left(\frac{1}{2}\right)^{2n} \leq \frac{1}{2}$. To see this, first note that when $n = 1$, this inequality holds. Now, suppose this inequality holds when $n = k$. Then, we have

$$\frac{(2(k+1))!}{(k+1)!(k+1)!}\left(\frac{1}{2}\right)^{2(k+1)} = \frac{(2k)!}{k!k!}\left(\frac{1}{2}\right)^{2k} \times \frac{(2k+1)(2k+2)}{(k+1)^2}\frac{1}{2^2}$$

$$\leq \frac{1}{2} \times \frac{2k+1}{2k+2}\frac{2k+2}{2k+2} \leq \frac{1}{2}.$$

Hence, we have

$$h'(0) \leq -(n+1) + (2n+1)\frac{1}{2} = -\frac{1}{2}.$$

Note also that $h(0) = 2$. Thus, there is $\bar{\varepsilon} > 0$ such that for all $\varepsilon < \bar{\varepsilon}$, $h(\varepsilon) < 2 - \frac{\varepsilon}{3}$. This implies that for all $\varepsilon < \bar{\varepsilon}$,

$$C(\varepsilon) - RV(\varepsilon) \geq C(\varepsilon) - \overline{RV}(\varepsilon) > 1 - \frac{1}{2}(2 - \frac{\varepsilon}{3}) - O(\varepsilon^2) = \frac{\varepsilon}{6} - O(\varepsilon^2).$$

Hence, there is $\hat{\varepsilon} > 0$ such that for all $\varepsilon < \hat{\varepsilon}$, $C(\varepsilon) - RV(\varepsilon) > 0$, and thus $C(\varepsilon) > RV(\varepsilon)$. This completes the proof. $\square$

A.11   Proof of Theorem 7

Proof. Fix a sequence $S = (X_1, \ldots, X_k)$ such that X_k appears exactly $n + 1$ times. Consider the consistent strategy profile σ satisfying the following conditions:

1. $\sigma_i(m, \emptyset)(X_i) = 1$ if $m = |\{X_l = A | 1 \leq l \leq i - 1\}|$.

2. $\sigma_i(m, \emptyset)(A) = 1$ if $m > |\{X_l = A | 1 \leq l \leq i - 1\}|$.

3. $\sigma_i(m, \emptyset)(B) = 1$ if $m < |\{X_l = A | 1 \leq l \leq i - 1\}|$.

Now we check incentives. First, voters who have received a signal take a best response given Cases 2 and 3 above.

Second, under any histories described in Case 2 above, by Bayes rule, i has a belief that the state is α. Hence, it is straightforward to see that i is taking a best response. A symmetric argument holds for Case 3 as well.

Third, under any histories described in Case 1 above, without loss of generality, let $X_k = A$. Suppose that, in the default sequence, there have been s_A times of A and s_B times of B, and there will be s^A times of A and s^B times of B. Voter i's posterior

on state α is then
$$P(\alpha) = \frac{(1-\varepsilon)^{s_B+1}}{(1-\varepsilon)^{s_A+1} + (1-\varepsilon)^{s_B+1}}. \tag{8}$$

Case 1: Suppose that $X_i = A$. Then, if i plays A, her payoff is

$$P(\alpha) \cdot 1 + (1 - P(\alpha))(1 - (1-\varepsilon)^{s^A}) = 1 - (1 - P(\alpha))(1-\varepsilon)^{s^A}.$$

If, instead, i plays B, then her payoff is at most:

$$P(\alpha)(1 - (1-\varepsilon)^{s^B}) + (1 - P(\alpha)) \cdot 1 = 1 - P(\alpha)(1-\varepsilon)^{s^B}$$

because, for the outcome to be A, there must be at least one time in the future when a voter who is supposed to play B in the default sequence receives signal a (and thus votes for A).

The former payoff is no less than the latter upper bound if and only if

$$(1 - P(\alpha))(1-\varepsilon)^{s^A} \le P(\alpha)(1-\varepsilon)^{s^B}$$

which, by (8), is equivalent to:

$$(1-\varepsilon)^{s_A+1}(1-\varepsilon)^{s^A} \le (1-\varepsilon)^{s_B+1}(1-\varepsilon)^{s^B},$$

or $s_A + s^A \ge s_B + s^B$.

Now, notice that, since S is the default sequence that ends with A and i plays A in S, we have that $s_A + 1 + s^A = n + 1$ and $s_B + s^B \le n$. Hence, we indeed have $s_A + s^A \ge s_B + s^B$, completing the proof for this case.

Case 2: Suppose that $X_i = B$. Then, if i plays B, her payoff is

$$P(\alpha) \cdot 1 + (1 - P(\alpha))(1 - (1-\varepsilon)^{s^A})$$

as in Case 1. If, instead, i plays A, then her payoff is at most:

$$P(\alpha) \cdot 1 + (1 - P(\alpha))(1 - (1-\varepsilon)^{s^A})$$

because, for the outcome to be B, there must be at least two times in the future when a voter who is supposed to play A in the default sequence receives signal b (and thus

votes for B). This event is less likely than the event in which there is at least one time in the future when a voter who is supposed to play A in the default sequence receives signal b (and thus votes for B). This latter event happens with probability at most $1 - (1 - \varepsilon)^{s^A}$. Now, since the former payoff is equal to the latter upper bound, it is indeed voter i's best response to play B. $\qquad\square$