# Naive and Sophisticated Choices under Intransitive Indifference

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**Abstract**

We define a sophisticated choice rule given underlying preferences that involve intransitive indifference, and compare it to its "naive" counterpart standard in the literature. The sophisticated choice breaks a tie between two alternatives using a third one to which the two are compared differently. We show that the definition of sophisticated choices captures the behavior of fully sophisticated decision makers, and that naive and sophisticated decision makers behave differently whenever intransitivity matters. The result fails if the data available to the analyst are incomplete.

Keywords: Intransitive indifference; WARP; sophistication.

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# 1 Introduction

The importance of intransitive indifference is recognized as early as Wiener (1914, 1915) who modeled such preferences by using binary relations called "interval order" and "semiorder."[1] The idea is that the decision maker may not perceive a small difference in utility while she may be able to distinguish between two elements that are far apart from each other. For example, the decision maker may be indifferent between $x$ and $y$ and between $y$ and $z$, while she strictly prefers $z$ to $x$. This may happen when $x$ and $y$ are close in utility and $y$ and $z$ are close, while $x$ and $z$ are far enough apart from each other so the decision maker can distinguish between these two.

This paper studies choice behaviors given such a general class of underlying preferences. We focus on two ways of postulating choice behaviors—that we call naive and sophisticated choices—and characterize the difference between these two.

The naive choice rule refers to the standard choice correspondence: it returns, for each choice set, its elements that are not strictly preferred by any other element in the set. In the above example, this corresponds to the choice of $\{x, y\}$ from the set $\{x, y\}$, and $\{y, z\}$ from the set $\{x, y, z\}$. Since $x$ is chosen in the presence of $y$ in the first problem while it is not chosen in the second problem in which $y$ is chosen, this choice rule does not satisfy the weak axiom of revealed preferences (WARP). Jamison and Lau (1973), Fishburn (1975), and Aleskerov, Bouyssou, and Monjardet (2007) axiomatized and discussed in a great depth this choice correspondence under various restrictions on underlying preferences.[2]

The basic idea of sophisticated choice goes back to Luce's (1956) definition, although he did not use it to model a choice rule but to characterize semiorder preferences. Since

[1] Preferences are represented by an interval order if there exist a utility level and the length of an interval for each alternative such that the decision maker strictly prefers $y$ to $x$ if and only if the utility from $y$ is strictly greater than the utility from $x$ plus the length of the interval for $x$. Preferences are represented by a semiorder if it is an interval order where the lengths of intervals are always a positive constant, say 1.

[2] The definition of naive choice is equivalent to what Aleskerov, Bouyssou, and Monjardet (2007) call choice. The sole reason that we added the quantifier "naive" is to make the comparison with the "sophisticated" one transparent, and it has nothing to do with our normative stance, if any.

then it does not seem to have attracted as much attention in the literature as the naive one. The sophistication criterion breaks a tie between two alternatives using a third one to which the two are compared differently. It is captured in the following inference that the decision maker could draw:[3]

> I am given a set $\{x, y, z\}$. I know that I'm indifferent between $x$ and $y$, and between $y$ and $z$, and that I strictly prefer $z$ to $x$. Since I strictly prefer $z$ to $x$, I have no reason to choose $x$. The remaining is $y$ and $z$, and I'm indifferent between these two. So I guess I choose $y$ and $z$ from this set... But wait, I know $y$ and $z$ are different in the ways they are compared to $x$. I strictly prefer $z$ to $x$, while I'm indifferent between $y$ and $x$. Although I do not see a difference between $y$ and $z$ probably because they are too similar, this should suggest that $z$ gives me a better utility than $y$. Hence I choose only $z$ from $\{x, y, z\}$.

Note that this decision maker's choice behavior fails WARP, as her choice would be $\{y, z\}$ if given $\{y, z\}$, as there is nothing in $\{y, z\}$ that discriminates $z$ from $y$. The addition of $x$ to the choice set gives the decision maker a way to discriminate $z$ from $y$.

The main theorem of this paper gives an understanding of the difference between the naive and sophisticated choice rules. It shows that if a choice correspondence is a sophisticated choice as well as a naive choice, then it satisfies WARP. That is, the way the sophisticated choice deviates from WARP is always different from the way that the naive choice deviates from it.[4] In other words, naive and sophisticated choices are different whenever intransitivity matters. The result implies that one should observe at most one kind of choice behavior, so this serves as a guide for identifying when each of these choice rules is relevant.

[3] In Remark 1, we provide more discussion on why and when this type of inference may make sense.

[4] This is a necessary condition for a choice to be sophisticated. Complete characterization is highly nontrivial and awaits a future research.

## 2 Sophisticated Preferences

Let $X$ be a (finite of infinite) set of elements, and $\succ$ be a binary relation over $X$, interpreted as strict preferences of the decision maker (i.e. $y \succ x$ means the decision maker strictly prefers $y$ to $x$).[5] We write $x \sim y$ if and only if $x \not\succ y$ and $y \not\succ x$.

In general, $\sim$ may not be transitive. The next definition captures the idea of the sophistication criterion discussed in the Introduction.[6]

**Definition 1.** For any $A \subseteq X$ and $x, y \in A$, we write $y \succ_A x$ if at least one of the following statements is true:

1. $y \succ x$;
2. there exists $w \in A$ such that $x \not\succ w$ and $y \succ w$;
3. there exists $z \in A$ such that $z \succ x$ and $z \not\succ y$.

Notice that in parts 2 and 3 of the above definition, $w$ and $z$ are taken from the given set $A$. As we have discussed, this is why the sophisticated choice may fail WARP.[7] We write $x \sim_A y$ if and only if $x \not\succ_A y$ and $y \not\succ_A x$.

Since all the alternatives that appear in Definition 1 are elements of $A$, we can define the sophistication criterion in Definition 1 for a binary relationship defined over $A \subseteq X$ in the same way (instead of defining it for a relation defined over $X$). We denote by $(\succ_A)_A$ the binary relation that emerges as a result of the sophistication criterion for $\succ_A$.

The following is the first result of this paper.

**Theorem 1.** *For any* $A \subseteq X$, $(\succ_A)_A = \succ_A$.

[5] At this point, we only require $\succ$ to be a binary relation, so for example it may not be acyclic or asymmetric or even irreflexive. As will be clear, our results hold even when one restricts attention to binary relations that satisfy these properties.

[6] In a different context from ours, an independent work by Frick (2015) also considers a rule that generates from a given preference relation a new preference relation, which generates more strict preferences than our sophistication criterion.

[7] Luce (1956) defines a similar preference relation (replacing $x \not\succ w$ and $z \not\succ y$ in Definition 1 with $x \sim w$ and $z \sim y$, respectively), but does not study the context-dependence of the choice, i.e., he only considers $\succ_X$.

*Proof.* Fix $A \subseteq X$ with $x, y, w \in A$, and suppose that $x \not\succ_A w$ and $y \succ_A w$. We show that $y \succ_A x$. The following are the possibilities. (i) $x \not\succ w$ and $y \succ w$, (ii) there exists $z \in A$ such that $x \not\succ w$ and $w \not\succ z$ and $y \succ z$, and (iii) there exists $z \in A$ such that $x \not\succ w$ and $z \succ w$ and $z \not\succ y$.

In case (i), $y \succ_A x$ by the definition of $\succ_A$. In case (ii), if $x \not\succ z$ then $y \succ_A z$. If $x \succ z$ then $x \succ_A w$, contradicting to the starting assumption. In case (iii), if $z \not\succ x$ then $x \succ_A w$, contradicting to the starting assumption. If $z \succ x$ then $y \succ_A x$.

Next, fix $A \subseteq X$ with $x, y, z \in A$, and suppose that $z \succ_A x$ and $z \not\succ_A y$. We show that $y \succ_A x$. The following are the possibilities. (i) $z \succ x$ and $z \not\succ y$, (ii) there exists $w \in A$ such that $x \not\succ w$ and $z \succ w$ and $z \not\succ y$, and (iii) there exists $w \in A$ such that $w \succ x$ and $w \not\succ z$ and $z \not\succ y$.

In case (i), $y \succ_A x$ by the definition of $\succ_A$. In case (ii), if $y \not\succ w$ then $z \succ_A y$, contradicting to the starting assumption. If $y \succ w$ then $y \succ_A x$. In case (iii), if $w \not\succ y$ then $y \succ_A x$. If $w \succ y$ then $z \succ_A y$, contradicting to the starting assumption. ■

The theorem implies that one cannot "further infer" her preferences beyond $\succ_A$. Thus, to capture the idea of the sophistication criterion described in the Introduction, it is enough to consider the relation $\succ_A$. In other words, our definition of sophisticated choices captures the behavior of fully sophisticated decision makers.

**Remark 1.** The degree to which sophisticated choices are reasonable may depend on various issues, such as the decision maker's intelligence, time to make a decision, and perhaps importantly the regularity imposed on the underlying preferences (in an extreme case, for example, if the preferences are cyclic then neither criterion might make so much sense). Although it is not the purpose of this paper to identify when these choice rules are relevant, we provide two arguments surrounding this theme. First, one interpretation of intransitive indifference is the idea of "just noticeable difference" that goes back to Luce (1956) in which he discusses an example of sugar being added to cups of coffee little by little, and the decision maker is indifferent between "adjacent" treatments but demonstrate strict preferences when a sufficient amount of sugar is added. This example is based purely on the decision maker's physiological limitation, and the reason for intransitivity would be best seen as orthogonal to the level of her intelligence. This suggests that it can make sense to consider sophistication even when preferences demonstrate intransitive indifference. Second, in the Appendix we provide a characterization of underlying preferences such that the induced sophisticated choice correspondence is nonempty. Assuming that $X$ is finite the underlying preferences that induce nonempty sophisticated choices turn out to be what is called an interval order (Fishburn, 1970). ■

# 3 Sophisticated Choice

Let a nonempty set $\mathcal{D} \subseteq 2^X \setminus \{\varnothing\}$ be the data available to the analyst. A choice correspondence, or simply **choice**, is a map $C : \mathcal{D} \to 2^X$ that assigns to each element of $\mathcal{D}$ a subset of that set. We say that the data are **complete** if $\mathcal{D} = 2^X \setminus \{\varnothing\}$, and that they are **incomplete** otherwise.

Given preferences $\succ$, we postulate two choice rules. For all $A \in \mathcal{D}$, let

$$C^N(A, \succ) = \{x \in A| \not\exists y \text{ s.t. } y \succ x\}$$

and

$$C^S(A, \succ) = \{x \in A| \not\exists y \text{ s.t. } y \succ_A x\}.$$

**Definition 2.** A choice correspondence $C$ is a **naive choice** if there exists $\succ$ over $X$ such that $C = C^N(\cdot, \succ)$.

**Definition 3.** A choice correspondence $C$ is a **sophisticated choice** if there exists $\succ$ over $X$ such that $C = C^S(\cdot, \succ)$.

As the example in the Introduction makes clear, sophisticated choices are not special cases of naive choices.[8] The reason that we use $\succ_A$ to define sophisticated choice is precisely Theorem 1: Given the set $A$, there is nothing that the decision maker can infer about her utility beyond $\succ_A$.

**Definition 4.** A choice correspondence $C$ satisfies the **weak axiom of revealed preferences**, or **WARP**, if

$$\exists A \in \mathcal{D} \text{ such that } x, y \in A \text{ and } x \in C(A) \Longrightarrow$$

$$\forall B \subseteq X \text{ such that } x, y \in B, \quad y \in C(B) \text{ implies } x \in C(B).$$

Implicit in Jamison and Lau (1973) and Fishburn (1975) is that a naive choice does not satisfy WARP. Since the binary relation $\succ_A$ depends on the given set $A$, sophisticated choice may not satisfy WARP either, as suggested in the Introduction. This leads us to the question of how this sophisticated choice deviates from WARP, compared to the way the "naive" counterpart deviates from it.

**Theorem 2.** *Suppose that the data are complete. Then, a nonempty choice correspondence $C$ is both naive and sophisticated choices if and only if it satisfies WARP.*[9] *That is, there exist $\succ$ and $\succ'$ such that $C = C^N(\cdot, \succ) = C^S(\cdot, \succ')$ if and only if $C$ satisfies WARP.*

**Remark 2.** Given the non-emptiness of the choice correspondence $C$, $\succ$ and $\succ'$ in the statement of the theorem must be equal, as the data are complete so in particular they include all the binary sets. Thus even if we replace $\succ'$ with $\succ$ in the statement, there is no loss of generality. In the statement of the theorem we postulated the two preferences in order to make transparent the comparison with the case with incomplete data. For the same reason, in the proof, we did not use the fact that the two preferences must coincide

[8] Since a sophisticated choice always returns a smaller set of alternatives than the naive counterpart does under the same underlying preferences, the idea is similar to that of Subiza and Peris (2000) who study selection of elements among the ones that the naive choices return. Lombardi (2008, 2009) also proposes choice rules that are based on similar selection criteria.

[9] In the Appendix we provide a counterexample that shows that the conclusion of this theorem fails if $C$ can be empty.

unless necessary. $\blacksquare$

*Proof.*

**"If" part:**

This part is straightforward: Note that $C(A) \neq \varnothing$ for all nonempty $A \subseteq X$. With this assumption, WARP implies that there exists an asymmetric and negatively transitive binary relation $\succeq$, hence in particular $\sim$ is transitive.[10] This implies that $\succ_A$ for any nonempty $A \subseteq X$ coincides with $\succ$, hence we have the desired result.

**"Only if" part:**

Suppose the contrary, i.e. that $C$ does not satisfy WARP but it is naive and sophisticated choices. Let $\succ^N$ and $\succ^S$ be binary relations with which $C$ is naive and sophisticated choices, respectively.

Since $C$ does not satisfy WARP, there exist $A, B \in 2^X$ and $x, y \in A, B$ such that (i) $x \in C(A)$ and (ii) $y \in C(B)$, but (iii) $x \notin C(B)$. (i) implies $y \not\succ^S x$, and (ii) implies $x \not\succ^S y$. Hence we have $x \sim^S y$. (iii) implies there exists $z$ such that $z \in C(B)$ and $z \succ^N x$. Hence we must have $C(\{x, z\}) = \{z\}$, as $z \in C(B)$ implies $z \not\succ^N z$. This implies $z \succ^S x$ since (i) implies $x \not\succ^S x$. Together we have $x \sim^S y$ and $z \succ^S x$, which imply $z \succ^S_B y$. Hence $y \notin C(B)$ holds, a contradiction. $\blacksquare$

The theorem says that the way sophisticated choice deviates from the standard choice behavior is always different from the way the "naive" counterpart deviates from it. Thus the naive and sophisticated choices have distinguishable choice implications as long as intransitivity matters- i.e. as long as WARP is violated.

As we noted, the two preference relations postulated in the theorem, $\succ$ and $\succ'$, are identical if $C^N = C^S$, as the data include all the binary choice problems. When the data are not complete, however, $C^N = C^S$ may not hold. It turns out that in such a case the conclusion of the theorem is not true. We describe this in Example 1. Next we turn to a

[10] See Arrow (1959).

positive result, stating that if we do restrict to the same preferences, then the conclusion of the theorem still holds.

To see the exact step of the proof where we use the assumption that all the data are available, notice that in the proof we concluded from $z \succ^N x$ that $z \succ^S x$, via the conclusion that $C(\{x, z\}) = \{z\}$.[11] For this we needed that $\{x, z\}$ is in our data. Of course we do not need to go via this conclusion about choice when we already assume $\succ^N = \succ^S$. This is why our positive result follows, and Theorem 3 summarizes this result. This also shows that if $\mathcal{D}$ includes all the binary choice problems then the conclusion of the theorem still holds. If the binary problem is missing from the data and if we do not assume $\succ^N$ matches with $\succ^S$, we cannot conclude anything from that, and consequently the logic fails. The counterexample exploits this point.

**Example 1.** Suppose $X = \{w, x, y, z\}$, $\mathcal{D} = \{\{x, y\}, \{w, x, y, z\}\}$, $C(\{x, y\}) = \{x, y\}$, and $C(\{w, x, y, z\}) = \{y, z\}$. Notice that the data that the analyst has are strictly smaller than $2^X \setminus \{\varnothing\}$. Since $x$ is chosen in the presence of $y$ given $\{x, y\}$ while it is not chosen when $y$ is chosen from $\{w, x, y, z\}$, WARP is violated. However we show that this choice correspondence is a naive choice as well as a sophisticated choice. To see this, we construct preferences with which $C$ is naive and sophisticated choices. Consider the semiorder preference relation represented by the following specification of utility function $u$ and the rule $x \succ y$ if and only if $u(x) - y(y) > 1$, and the one represented by $v$ and the rule $x \succ y$ if and only if $v(x) - v(y) > 1$:

$$u(w) = 0,\ u(x) = 1,\ u(y) = 2,\ u(z) = 3.$$

$$v(w) = 0,\ v(x) = 1,\ v(y) = 2,\ v(z) = 2.$$

It is straightforward to check that a naive choice with preferences represented by $u$ and a sophisticated choice with preferences represented by $v$ both give the choice $C$. We

[11] This is the only step we used this assumption.

note that, given Theorem 3 below, it is a necessary feature of this example that the two utility functions, $u$ and $v$, are different. The example is not incompatible with Theorem 2 because if the data were complete, then the choice from the set $\{x, z\}$ would be different given the preferences represented by $u$ and $v$, so the premise of the theorem does not hold. ■

**Theorem 3.** *Fix the data, complete or incomplete. Then, choice correspondence $C$ is both naive and sophisticated choices under the common preference relation if and only if it satisfies WARP. That is, there exists $\succ$ such that $C = C^N(\cdot, \succ) = C^S(\cdot, \succ)$ if and only if $C$ satisfies WARP.*

We omit the proof because it is immediate from that of Theorem 2.

# A Appendix: Non-Emptiness of Sophisticated Choices

The following shows the claim made in Remark 1.

Recall from Fishburn (1970) that $\succ$ is an interval order if it is irreflexive and $z \succ y$ and $x \succ w$ imply $z \succ x$ or $x \succ y$.

**Theorem 4.** *Suppose $X$ is finite. $\succ$ is an interval order over $X$ if and only if $C^S(A, \succ) \neq \varnothing$ for all $A \subseteq X$.*

*Proof.*

**"Only if" part:**

Suppose that $\succ$ is an interval order. Fishburn (1970) shows that there then must exist two functions $u_H : X \to \mathbb{R}$ and $u_L : X \to \mathbb{R}$ such that $u_H(x) \geq u_L(x)$ for all $x \in X$ and for any $x, y \in X$, $x \succ y$ if and only if $u_L(x) > u_H(y)$. Fix $u_H$ and $u_L$. For each $A \subseteq X$, define $x_A$ to be an arbitrary element in $\arg\max_{x \in A} u_L(x)$. Such $x_A$ exists because $A$ is finite (recall that $X$ is finite and $A \subseteq X$).

First, we note that there does not exist a $y \in A$ such that $y \succ x_A$, because $y \succ x_A$ would imply $u_L(y) > u_H(x_A)$. But since $u_H(x_A) \geq u_L(x_A)$, we then would obtain $u_L(y) > u_L(x_A)$, which contradicts the definition of $x_A$.

Second suppose that there exist $y$ and $w$ such that $x_A \not\succ w$ and $y \succ w$. The former implies that $u_L(y) > u_H(w)$, and the latter implies that $u_H(w) \geq u_L(x_A)$. Combining, we obtain $u_L(y) > u_L(x_A)$, contradicting the definition of $x_A$.

Finally, suppose that there exist $y$ and $z$ such that $z \succ x_A$ and $z \not\succ y$. But this is impossible because there cannot exist such $z$ that satisfies $z \succ x_A$ as we have already shown.

**"If" part:**

Suppose that $\succ$ is not an interval order. Then, either (i) there exists $x$ such that $x \succ x$, or (ii) there exist $w$, $x$, $y$, and $z$ such that $z \succ y$, $x \succ w$, $z \not\succ w$, and $x \not\succ y$.

In case (i), $C^S(\{x\}, \succ) = \varnothing$ holds because $x \succ x$ implies $x \succ_{\{x\}} x$.

In case (ii), (ii)-(a) $z \succ y$ and $x \not\succ y$ imply $y \succ_{\{x,y,z,w\}} x$, (ii)-(b) $z \succ y$ implies $z \succ_{\{x,y,z,w\}} y$, (ii)-(c) $x \succ w$ and $z \not\succ w$ imply $x \succ_{\{x,y,z,w\}} z$, and (ii)-(d) $x \succ w$ implies $x \succ_{\{x,y,z,w\}} w$. Hence, $C^S(\{w,x,y,z\}, \succ) = \varnothing$ holds because no element in $\{w,x,y,z\}$ is undominated under $\succ_{\{w,x,y,z\}}$. ■

Note that Luce (1956) proves equivalence between the underlying preferences being a semiorder, and it being transitive as well as his induced preferences (defined in footnote 7) being a weak order. The induced preferences can imply a nonempty choice without being a weak order, so our result is not implied by his result.[12]

The "if" part of Theorem 2 fails if we do not impose non-emptiness of the choice correspondence $C$. Consider the following example:

**Example 2.** Suppose that $X = \{x, y, z\}$, $C(\{x, y, z\}) = \{y, z\}$, $C(\{y, z\}) = \varnothing$, $C(\{x, y\}) = \{y\}$, $C(\{x, z\}) = \{z\}$. This is a choice correspondence that chooses elements that are

[12] For example, consider $X = \{w, x, y, z\}$ with the underlying preferences $z \succ y$, $y \succ x$, and $z \succ x$ (with other relations being indifference). $\succ_X$ is not a weak order because $y \succ w$ and $w \succ y$, but $C^S(X) = \{z\} \neq \varnothing$.

strictly preferred to some element in the given choice problem, with $y \succ x$ and $z \succ x$. WARP is satisfied, but this is neither a naive nor a sophisticated choice: $C(\{y,z\}) = \varnothing$ implies the decision maker strictly prefers some element in $\{y,z\}$ to each of $y$ and $z$ in either choice rule, but these two imply that $C(\{x,y,z\})$ cannot include either $y$ or $z$.[13] ■

**Remark 3.** Let us be clear on the limitation of our approach in light of both necessity and sufficiency shown in Theorem 4. First, a sophisticated choice may be empty if the given preference relation is not an interval order. This implies that the sophistication criterion fails to capture certain anomalies. For example, consider the attraction effect (Doyle et al., 1999), where $X = \{x^1, x^2, y^1, y^2\}$, $x^1$ dominates $x^2$ with respect to all characteristics, $y^1$ dominates $y^2$ with respect to all characteristics, and there is no clear comparison between $x^i$ and $y^j$ for each pair $(i,j) \in \{1,2\}^2$. Then, we would have $x^1 \succ x^2$, $y^1 \succ y^2$, and $x^i \sim y^j$ for each pair $(i,j) \in \{1,2\}^2$. The typical story of the attraction effect would suggest that $A := \{x^1, y^2\}$ is chosen from $A$, $\{x^1\}$ is chosen from $B := \{x^1, x^2, y^2\}$ and $\{x^1, x^2\}$ is chosen from $X$. However, we would have $C^S(A, \succ) = A$, $C^S(B, \succ) = \{x^1\}$ and $C^S(X, \succ) = \varnothing$, only partially capturing the attraction effect. Second, the non-emptiness of a choice function may not be enough to guarantee the reasonability of the resulting choice. To see this in an example, recall from Fishburn (1970) that $\succ$ is an interval order if and only if there exists a pair of functions $u : X \to \mathbb{R}$ and $\rho : X \to \mathbb{R}_+$ such that $y \succ x$ is equivalent to $u(y) > u(x) + \rho(x)$. Here, the interpretation is that each $x \in X$ is assigned an interval $[u(x), u(x) + \rho(x)]$, and the decision maker strictly prefers $y$ to $x$ if and only if the upper bound of the internal for $x$ is strictly lower than the lower bound of the internal for $y$. Now, consider $x$, $y$, and $z$ with associated intervals $[1,2]$, $[0,5]$, and $[3,4]$. The resulting preference relation is $x \sim y$, $y \sim z$, and $z \succ x$, so we have $C^S(\{x,y,z\}, \succ) = z$. However, without further contextual information apart from that of the intervals, it is not clear if we should conclude that $z$ should be chosen. Such a concern would be less of an issue if the lengths of the intervals are assumed to be common

[13] As is clear, this example works even if we assume $\succ$ to be irreflexive. When $\succ$ can be reflexive, a simpler example can be obtained: $X = \{x,y\}$, $C(\{x,y\}) = \{x,y\}$, $C(\{x\}) = \varnothing$, and $C(\{y\}) = \varnothing$.

across all alternatives (which is equivalent to assuming that $\succ$ is a semiorder) and the interpretation of the intervals is the idea of "just noticeable difference" as motivated in the Introduction.[14] Again, it is not our purpose to identify when our sophistication criterion is relevant. Also, we do not intend to capture all possible anomalies, and we hope the above two points deepen the understating of our sophistication criterion.

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[14] An alternative interpretation of interval orders is that the decision maker is uncertain about the utility from each alternative $x$ and believes that it lies in the interval $[u(x), u(x) + \rho(x)]$.

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