Asynchronous Revision Games with Deadline: Unique Equilibrium in Coordination Games

Yuichiro Kamada and Takuo Sugaya*

June 21, 2010

Abstract

Players prepare their actions before they play a normal-form game at a predetermined deadline. In the preparation stage, each player stochastically receives opportunities to revise their actions, and a finally-revised action is played at the deadline. We show that (i) a strictly Pareto-dominant Nash equilibrium, if any, is the only equilibrium in the dynamic game if the normal form game is sufficiently close to a pure coordination game; and (ii) in "battle of the sexes" games, (ii-a) with a symmetric structure, the equilibrium payoff set is a full-dimensional subset of the feasible payoff set, but (ii-b) a slight asymmetry is enough to select a unique equilibrium.

Keywords: Revision games, finite horizon, equilibrium selection, asynchronous moves

JEL codes: C72, C73

*Kamada: Department of Economics, Harvard University, Cambridge, MA, 02138, e-mail: ykamada@fas.harvard.edu; Sugaya: Department of Economics, Princeton University, Princeton, NJ, 08544, e-mail: tsugaya@princeton.edu; We thank Dilip Abreu, Riccardo Calcagno, Gabriel Carrol, Sylvain Chassang, Drew Fudenberg, Michihiro Kandori, Fuhito Kojima, Barton Lipman, Stefano Lovo, Stephen Morris, Satoru Takahashi, and seminar participants at Research in Behavior in Games seminar at Harvard University and at Student Lunch Seminar in Economic Theory at Princeton University for helpful comments. In addition, Yuhta Ishii read through the previous version of this paper and gave us very detailed comments, which significantly improved the presentation of the paper.

1 Introduction

Strategic interactions often require the preparation of strategies in advance and even though players interact once and for all, there can be a long period for the preparation of strategies that influences the outcome. For example, consider two firms which have an investment opportunity. The date to make the actual investment is fixed and the same across the firms. However, before the actual investment, two firms need to make preparations: negotiating with banks to have enough liquidity, allocating agents working for the project, and so forth. It is natural to think that the investment is profitable for a firm if the opponent invests, and vice versa. Moreover, while the firms make the preparation for the investments, it is often the case that firms cannot always change their past preparations. That is, they have several constraints for changing past preparations, such as administrative procedures, obligations to other projects, and so forth. Rather, opportunities to revise preparations may arrive stochastically over time.

Having stochastic opportunities for revisions allows players to credibly commit to the chosen actions. In the above example, a firm's decision to invest depends on the opponent's investment decision. If a firm makes preparations for the investment, there is a positive probability that it will not have any further opportunities to revise its decision leading the firm to invest, thus making investment more profitable for the opponent.

The present paper models such situations as follows: A normal-form game is played once and for all at a predetermined deadline, before which players have opportunities to revise their actions. These opportunities arrive stochastically and independently across players. To model the preparations of actions, we use the framework of a finite horizon version of "revision games," proposed by Kamada and Kandori (2009). In revision games, players prepare their actions at opportunities that arrive with a Poisson process in continuous time until a predetermined deadline. The actions prepared most recently at the deadline are played once and for all. They consider the limit as the length of the preparation stage goes to infinity, so that the probability of having many revision opportunities by the deadline is sufficiently high. This is equivalent to considering the limit as the arrival rate becomes very high for a fixed length of preparation stage. That is, they consider the situation in which players can revise their actions very frequently. We consider such a limit as well in this paper.

We show that, even when revision opportunities arrive with a very high frequency, the small possibility of not being able to further change the prepared action has surprisingly strong effects on the equilibrium selection: (i) if there exists a Nash equilibrium that strictly Pareto-dominates all other action profiles, then it is the only equilibrium of the corresponding revision game if the normal form game is sufficiently close to a pure coordination game (games in which all players' payoff functions are the same); and (ii) in 2×2 games with two strict pure Nash equilibria which are not Pareto-ranked, (ii-a) while with perfectly symmetric structure, the equilibrium payoff set is a full-dimensional subset of the feasible payoff set, (ii-b) a slight asymmetry is enough to select a unique equilibrium, which corresponds to the Nash equilibrium in the static game that gives the highest payoff to the "strong" player.

Calcagno and Lovo (2010) independently investigate essentially the same set of questions as ours with several key differences. Section 2 details its explanation and also provides a review of other related past works.

Let us briefly explain the intuition behind our result.[1] First, consider a game with a strictly Pareto-dominant Nash equilibrium. We show that once all players prepare the strictly Pareto-dominant Nash equilibrium action, they will not escape from that state. With this expectation, if a unilateral change of the preparation by player i can induce the strictly Pareto-dominant Nash equilibrium action profile, she will do so. In turn, player j whose unilateral change in the preparation can induce the situation where player i's unilateral change can induce the strictly Pareto-dominant Nash equilibrium actions, she will also do so if the deadline is far expecting that player i will go to the strictly Pareto dominant Nash equilibrium action profile. A nontrivial extension of this argument shows that the result holds for any number of players when the stage game is sufficiently close to a pure coordination game.

[1] A detailed explanation can be found in Section 6.

Let us move onto 2×2 games with two Pareto-unranked strict Nash equilibria. To simplify the explanation, consider the "battle of the sexes" games with the following payoff matrices:

	L	R
U	$2,1$	$0,0$
D	$0,0$	$1,2$

Symmetric "battle of the sexes" game

	L	R
U	$2+\epsilon,1$	$0,0$
D	$0,0$	$1,2$

Asymmetric "battle of the sexes" game ($\epsilon > 0$)

First, consider the left payoff matrix, the symmetric "battle of the sexes" game, and assume that arrival rates are homogeneous across players. We prove that the equilibrium payoff set is a full dimensional subset of the feasible payoff set, as depicted in Figure 2. To establish this result, and to compare it with that for the asymmetric "battle of the sexes" game, it is important to note that there exists what we call a *chicken race equilibrium*, in which players prepare (U, R) until some cutoff time, and gives in to play the other action after that cutoff. Since at the cutoff time players are indifferent between giving in to obtain the payoff of 1 and sticking to the original inefficient profile, each player expects a payoff of 1 from this equilibrium.[2] Notice that the payoff 1 corresponds to each player's "worse" Nash equilibrium payoff.

Next consider either the right payoff matrix, the asymmetric "battle of the sexes" game, with homogeneous arrival rates, or the symmetric "battle of the sexes" game with asymmetric arrival rates where player 1's arrival rate is smaller than player 2's. We prove that the only equilibrium payoff is the one that corresponds to the Nash equilibrium that the "strong" player (player 1) prefers (See Figure 3). Here, the meaning of "strong" is important. In the asymmetric-payoff case, the "strong" player is the one who expects more in his preferred Nash equilibrium, (U, L), than the opponent (player 2) does in the other pure Nash equilibrium, (D, R). In the asymmetric-arrival-rate case, the "strong" player is the one with the lower arrival rate of revisions, that is, the one who can more credibly commit to the currently

[2] There are also equilibria in which players stick to either of two Nash equilibria, and we prove that almost all points in the convex hull of these three payoff points are attainable in revision games.

prepared action. To see the reason for equilibrium selection, consider the "chicken race equilibrium" discussed in the previous paragraph. Because of the asymmetry, the two players have different cutoff times at which they become indifferent between giving in and sticking to (U, L). Specifically, the strong player 1 must have a cutoff closer to the deadline than that of the weak player 2. This implies that, in the chicken race equilibrium, if it exists, the strong player 1 expects strictly more than his "worse" Nash equilibrium payoff, (D, R), while the weak player 2 expects strictly less than her "worse" Nash equilibrium payoff, (U, L). Hence, the strong player 1 would not want to stick to the "worse" Nash equilibrium, (D, R), which rules out the possibility of sticking to (D, R). Also, the weak player 2 would not want to stick to the chicken race equilibrium, which rules out the existence of a chicken race equilibrium. Therefore the only possibility is the (U, L) profile. Thus a unique profile is selected.

The above discussion implies that the selected equilibrium is determined by a joint condition on the payoff structure and the arrival rate. In Section 5, we formally state the condition for the existence of the "strong" player whose preferred equilibrium in the stage game is selected. We will see that, for any 2×2 games with two strict pure strategy Pareto-unranked Nash equilibria, there is a nongeneric set of parameters where the equilibrium payoff set is a full-dimensional subset of the feasible payoff set. On the other hand, for any parameter in the complement of this nongeneric set, a unique equilibrium is selected. We provide a detailed explanation of the intuition in Section 6 in a simplified setting where the two Nash equilibria are absorbing states. Note that, although the intuition looks simple, proving that the equilibrium is unique is not an easy task. This is because no Nash action profile being a priori absorbing, when the deadline is far away, players might have incentives to move away from a Nash profile. The above explanation captures a part of this effect because player 1 does not want to stick to his "worse" Nash equilibrium (D, R). We will show that given player 1's optimal strategy in the subgame, even player 2 does not want to stick to this profile (which he likes the best) when the deadline is far away.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 introduces the model. Section 4 considers the game with a strictly Pareto-dominant Nash equilibrium and Section 5 considers 2×2 games with Pareto-unranked multiple equilibria. The basic logic is explained in Section 6. Section 7 concludes.

2 Literature Review

In many works on equilibrium selection, risk-dominant equilibria of Harsanyi and Selten (1988) are selected in 2×2 games. In our model, however, a different answer is obtained: a strictly Pareto-dominant Nash equilibrium is played even when it is risk-dominated if the normal form game is sufficiently close to a pure coordination game. Roughly speaking, since we assume perfect and complete information with nonanonymous players, there is only a very small "risk" of mis-coordination when the deadline is far. There are three lines of the literature in which risk-dominant equilibria are selected: models of global games, stochastic learning models with myopia, and models of perfect foresight dynamics.[3,4] Since the model of perfect foresight dynamics seem closely related to ours, let us discuss it here.

Perfect foresight dynamics, proposed by Matsui and Matsuyama (1994), are evolutionary models in which players are assumed to be patient and "foresighted," that is, they value the future payoffs and take best responses given (correct) beliefs about the future path of the play.[5] There is a continuum of agents who are randomly and anonymously matched over infinite horizon according to a Poisson process. Whenever they match, they play the stage game and receive the stage game payoff. They show that, when a Pareto-dominant

[3] The literature on global games was pioneered by Rubinstein (1981), and analyzed extensively in Carlsson and van Damme (1993), Morris and Shin (1998), and Sugaya and Takahashi (2009). They show that the lack of almost common knowledge due to incomplete information can select an equilibrium. The type of incomplete information they assume is absent in our model. Stochastic learning models with myopia are analyzed in Kandori, Mailath, and Rob (1993) and Young (1993). They consider a situation in which players interact repeatedly, and each player's action at each period is stochastically perturbed. The key difference between their assumptions and ours is that in their model players are myopic, while we assume that players take actions expecting the opponents' future moves. In addition, the game is repeated infinitely in their models, while the game is played once and for all in our model.

[4] As an exception, Young (1998) shows that in the context of contracting, his evolutionary model does not necessarily lead to risk-dominant equilibrium (p-dominant equilibrium in Morris, Rob and Shin (1995)). But he considers a large anonymous population of players and repeated interaction, so the context he focuses on is very different from the one we focus on.

[5] See also Oyama, Takahashi, and Hofbauer (2008).

equilibrium is not risk-dominant, there always exists a path that starts at a Pareto-dominant equilibrium and ends at a risk-dominant equilibrium, while the opposite path does not exist. In this sense they say that the latter is "selected." The assumptions of the continuum of agents and the flow payoffs are key to their result, and indeed we show in our model that if there exists only a finite number of agents, the Pareto-dominant equilibrium is selected when the game satisfies some regularity conditions.

As opposed to these results in the literature, some models select a strictly Pareto-dominant Nash equilibrium. Farrell and Saloner (1985) and Dutta (2003) are early works on this topic, and Takahashi (2005) proves such results in a very general context. Takahashi's (2005) condition for selecting a strictly Pareto-dominant Nash equilibrium is similar to ours: the stage game is sufficiently close to a pure coordination game. The difference of our model is that we assume the actual game is played once at the end.

The intuition behind the chicken race equilibrium is similar to the one for the "war of attrition." The war of attrition is analyzed in, among others, Abreu and Gul (2000) and Abreu and Pearce (2000). They consider the war of attrition problem in the context of bargaining. They show that if there is an "irrational type" with a positive probability, then agreement delays in equilibrium because rational players try to imitate the irrational types. Players give in at the point where imitation is no longer profitable. Although the structure of this equilibrium in the war of attrition is similar to the chicken race equilibrium, the focus of their work and ours is clearly different.

Our results crucially hinge on asynchronicity of the revision process. If the revision process were synchronous, the very same indeterminacy among multiple strict Nash equilibria would be present. There are a limited number of papers showing that asynchronous moves select an equilibrium. Lagunoff and Matsui (1997) show that in pure coordination games the Pareto-efficient outcome is chosen.[6] We, although in a slightly different context in which the game is played only once at a deadline, identify a more general sufficient condition to

[6] According to Dutta (1995), this result is due to the lack of full dimensionality of the feasible and individually rational payoff set. See also Lagunoff and Matsui (2001).

obtain the best outcome as a unique equilibrium.[7]

Several papers obtain sharper results by assuming the presence of switching cost when moves are asynchronous. Lipman and Wang (2000) consider a finite horizon repeated games with small switching costs, in which the per-period length (hence the per-period payoff) is very small relative to the costs. With the same stage game as we consider in this paper, they select a unique equilibrium. However, there are three important differences. First, as the frequency of revision increases, it is potentially possible in our model that players change their actions very often, while in Lipman and Wang (2000), the switching cost essentially precludes this possibility. Second, the game is repeatedly played in their model, while it is played only once at the deadline in our model. Hence, in particular, their model cannot have "chicken race equilibrium" type of strategies in equilibrium. Third, in their model, the prediction of the game is not robust to affine transformation of payoffs.[8] Hence, in some payoff specifications, their selection matches ours, while in other cases it does not. The reason for this scale-dependence is that the switching cost is directly payoff-relevant. In our case, detailed specifications of the preparation stage (such as the arrival rate) is not directly payoff-relevant, so our result is robust to the affine transformation of the payoffs.

Caruana and Einav (2008) consider a similar setting as ours: players play the actual game once and for all at the fixed date and before actually playing the game, they play an asynchronous move dynamic game to commit to the actions. Although players can change their previous commitment before the deadline, they have to pay a switching cost. They show that in generic 2×2 games there is a unique equilibrium, irrespective of the order and timing of moves and the specification of the protocol. Our paper is different from theirs in two aspects. First, in their model, the switching cost plays an important role to select an

[7] For asynchronicity with respect to the timing of signals in imperfect monitoring setting, see Fudenberg and Olszewski (2009).

[8] This nonrobustness is fine for their purpose since they focus on pointing out that the set of outcomes discontinuously changes by the introduction of a small switching cost. However, it would not be very ideal in our context because we focus on identifying *which* outcome is selected by the introduction of the preparation stage. For that purpose, we want to minimize the information necessary for the selection: each player's preference and arrival rate are sufficient information to identify the outcome. In particular, the information to compare different players' payoffs is not necessary.

equilibrium. On the other hand, in our model, players can revise the preparation without cost whenever they get the revision opportunity. Hence, with high Poisson arrival rate, players can change their actions frequently without paying cost. Second, in Caruana and Einav, the order of the movement at each date is predetermined. In our model, the opportunity of the movement is stochastic. With the symmetric arrival rate, we can consider the situation that at each date, each player can move equally likely.

Let us mention the emerging literature on "revision games." Kamada and Kandori (2009) introduce the model of revision games. They show that, among other things, non-Nash "cooperative" action profiles can be played at the deadline when a certain set of regulatory conditions are met. Hence their focus is on expanding the set of equilibria when the static Nash equilibrium is inefficient relative to non-Nash profiles.[9] We ask a very different question in this paper: we consider games with multiple efficient static Nash equilibria, and ask which of these equilibria is selected.[10]

In the context of revision games with a finite action set, Kamada and Sugaya (2010) consider a model of an election campaign with three possible actions (Left, Right, and Ambiguous). The main difference of this paper from their work is that they assume that once a candidate decides which of Left and Right to take, she cannot move away from that action. Thus the characterization of the equilibrium is substantially more difficult in the model of the present paper, because in our model an action a player has escaped from can be taken again by that player in the future.

Independent of the present paper, Calcagno and Lovo (2010) consider the equilibrium selection problem by considering a "preopening game." In the preopening game, the actual game is played once and for all at the predetermined opening time. Before the opening date, players continuously submit their action. The preopening game is different from the revision game in that players do not have to wait for a revision opportunity but can continuously submit their action. A submitted action has some chance to be the action at the opening time

[9] The possibility of cooperation in finite horizon in Kamada and Kandori (2009) is closely related to that of finitely repeated games with multiple Nash equilibria (Benoit and Krishna, 1985).

[10] See also Ambrus and Lu (2009) for a variant of revision games model of bargainig in which the game ends when an offer is accepted.

only if it is consistently submitted for some period with strictly positive length. While the two models are different, our results are consistent with theirs with the following key differences: First, they only consider two-player games, while our result for a Pareto-dominant Nash equilibrium covers n-player games. Second, we fully characterize the equilibrium payoff set for all nongeneric cases such as symmetric "battle of the sexes" games (the set is a full dimensional subset of the feasible payoff set), but they only prove that there is more than one equilibrium. Third, they obtain results on a model where the probabilities that the actions submitted at a given time will be the actions at the opening time are correlated among players, while we only consider the case with independent revision opportunities. This article is highly recommended for interested readers.

3 Model

We consider a normal-form game $\left(I, (X_i)_{i\in I}, (\pi_i)_{i\in I}\right)$ where I is the set of players, X_i is the finite set of player i's actions, and $\pi_i : \times_{j\in I} X_j \to \mathbb{R}$ is player i's utility function. Before players actually take actions, they need to "prepare" their actions. We model this situation as in Kamada and Kandori (2009): time is continuous, $-t \in [-T, 0]$ with $T = 1$, and the normal form game (referred to as a "component game") is played once and for all at time 0. The game proceeds as follows. First, at time -1, players simultaneously choose actions. Between time -1 and 0, each player i independently obtains opportunities to revise their prepared action according to a Poisson process with arrival rate λ_i with $\lambda_i > 0$. At $t = 0$, the action profile that has been prepared most recently by each player is actually taken and each player receives the payoff that corresponds to the payoff specification of the component game. Each player has perfect information about past events at any moment of time. No discounting is assumed, although this assumption does not change any of our results.[11]

We consider the limit of the set of subgame-perfect equilibrium payoffs of this game as the arrival rate λ_i goes to infinity. Let $\phi^{(\lambda_i)_{i\in I}}(\pi)$ be the set of subgame-perfect equilibrium

[11] Discounting only scales down the payoff at time 0.

payoffs given arrival rates $(\lambda_i)_{i \in I}$.

Definition 1 *A payoff set $S \subset \mathbb{R}$ is a* ***revision equilibrium payoff set*** *of π if $S = \phi^{(r_i)_{i\in I}}(\pi) := \lim_{\substack{\lambda_i \to \infty \text{ for all } i \\ \lambda_i / \Sigma_{j=1}^n \lambda_j = r_i \text{ for all } i}} \phi^{(\lambda_i)_{i\in I}}(\pi)$. If $\phi^{(r_i)_{i\in I}}(\pi)$ is a singleton, we say its element is a* ***revision equilibrium payoff****. The set of distributions over the action profiles that correspond to the revision equilibrium payoff set is a* ***revision equilibrium set****. If the revision equilibrium set is a singleton, we say its element is a* ***revision equilibrium****.*

That is, a revision equilibrium payoff set is the set of payoffs achievable by the revision game defined in this section. Note that $r_i = \lambda_i / \Sigma_{j=1}^n \lambda_j$ represents the relative frequency of the arrivals of the revision opportunities for player i. It will turn out in what follows that this set is often a singleton. The term "revision equilibrium payoff" is used for a convenient abbreviation that represents the element of such a singleton set. "Revision equilibrium set" and "revision equilibrium" are analogously defined.

Note well that whenever we refer to some action profile (resp. a payoff profile) as a revision equilibrium (resp. a revision equilibrium payoff), we implicitly mean that it is the unique element of the revision equilibrium set (resp. revision equilibrium payoff set).

In what follows, to simplify the exposition, we consider the limit as T goes to infinity, holding fixed the arrival rate profile $(\lambda_i)_{i\in I}$. This is equivalent to considering the limit as the arrival rates go to infinity, holding fixed the length of preparation stage. All the notions introduced in this section are invariant with respect to this modification.

4 Strictly Pareto-Dominant Nash Equilibrium in a Component Game

In this section we consider a component game with a Nash equilibrium that strictly Pareto-dominates all other action profiles.

Definition 2 *A strategy profile x^* is said to be a* ***strictly Pareto-dominant Nash equilibrium*** *if for all i and all $x \in X \equiv \times_{i\in I} X_i$ with $x \neq x^*$, $\pi_i(x_i^*) > \pi_i(x)$.*

Note that this condition is stronger than Pareto-ranked Nash equilibria since there may exist a non-equilibrium payoff profile which one player prefers most.

It is straightforward to show that, if the currently prepared action profile is the strictly Pareto-dominant equilibrium, then in any subgame perfect equilibrium, no one switches actions from that state (**Lemma 9** in the Appendix). Hence, in particular, each player taking the action prescribed by x^* at the initial date $-T$ and continuing to take that action for all time $-t$ on the equilibrium path is always an equilibrium.

However, this does not immediately imply that x^* is the *unique* outcome in the revision game. Below we show that it is indeed the unique outcome in some cases but not in others. To establish the result, let us introduce the following definition.

Definition 3 *Fix a component game g with a strictly Pareto-dominant Nash equilibrium, x^*. g is said to be a K-**coordination game** if for any $i, j \in I$ and $x \in X$,*

$$\frac{\pi_i(x^*) - \pi_i(x)}{\pi_i(x^*) - \underline{\pi}_i} \leq K \frac{\pi_j(x^*) - \pi_j(x)}{\pi_j(x^*) - \underline{\pi}_j}$$

with $\underline{\pi}_i = \min_x \pi_i(x)$.

Note that K measures how close a component game g is to a pure coordination game. The minimum of K is 1 when the game is a pure coordination game, where players have exactly the same payoff functions.

Proposition 4 *Suppose that x^* is a strictly Pareto-dominant Nash equilibrium and there exists K such that g is a K-coordination game and*

$$\min_{i,j \in I, i \neq j} (r_i r_j) + \left(1 - \min_{i,j \in I, i \neq j} (r_i + r_j)\right)\left(1 - K\left(1 - \min_{i \in I} r_i\right)\right) > 0. \tag{1}$$

Then, x^* is the revision equilibrium and thus the unique equilibrium outcome.

Before discussing the intuition behind the proof, let us mention several remarks. First, it is worthwhile to make comments on Condition (1). The smaller K is, the more likely

Condition (1) to be satisfied. Especially, if the game is pure-coordination, that is, if $K = 1$, then (1) is always satisfied. In addition, for fixed K and number of players, if the relative frequency r_i is more equally distributed, (1) is more likely to be satisfied. Note also that if we decrease the number of players, fixing $K > 1$, (1) will be satisfied for sufficiently small number of players. Especially, for two-player games, since $(1 - \min_{i,j \in I, i \neq j} (r_i + r_j)) = 0$, (1) is automatically satisfied. Together with the fact that the existence of two strict Pareto-ranked Nash equilibria in 2×2 games implies the existence of strictly Pareto-dominant Nash equilibrium, this gives us the following corollary.[12]

Corollary 5 *For 2×2 games with two strict Pareto-ranked Nash equilibria, the Pareto efficient Nash equilibrium is the unique revision equilibrium.*

Second, notice that the revision game selects the strictly Pareto-dominant Nash equilibrium even if it is risk-dominated by another Nash equilibrium if the component game is sufficiently close to a pure coordination game (this condition is satisfied for all the two-player games). The key is that, if the remaining time is sufficiently long, since it is almost common knowledge that the opponent will move to the Pareto-dominant Nash equilibrium afterwards if (1) is satisfied, the risk of mis-coordination can be arbitrarily small.

Finally, notice also that we allow for g to be different from the pure-coordination game as long as (1) is satisfied. This result is in a stark difference from Lagunoff and Matsui (1997), in which they need to require that the game is of pure coordination, as otherwise their result would not hold (Yoon, 2001).

An intuitive explanation goes as follows. Firstly, we show that once all players prepare the strictly Pareto-dominant Nash equilibrium actions, they will not escape from that state. With this expectation, if a unilateral change of the preparation by player i can induce the strictly Pareto-dominant Nash equilibrium, she will do so. In turn, player j whose unilateral change in the preparation can induce the situation where player i's unilateral change can induce the strictly Pareto-dominant Nash equilibrium, will also do so if the deadline is

[12] Note that Kamada and Kandori (2009) proves that if each player has a dominant action when the action space is finite, it is played in asynchronous revision games.

far expecting that player i will go to the strictly Pareto-dominant Nash equilibrium. By induction, we can show that the strictly Pareto-dominant Nash equilibrium will be selected.

The formal proof requires more arguments since player j takes into account the possibility that player k currently taking x_k^* will have an opportunity to revise her action before player i and will switch to some other action.[13] If the game is close to a pure coordination game, then player k does not have an incentive to take an action different from x_k^* since it hurts player k herself. Otherwise, this might induce the outcome different from x^*. This is exactly why we need Condition (1).

Consider the following game: $g = \left(\{1,2,3\}, (\{a_i, b_i\})_{i\in\{1,2,3\}}, (\pi_i)_{i\in\{1,2,3\}}\right)$ with

x_1, x_2, x_3	π_1, π_2, π_3
a_1, a_2, a_3	$(1, 1, 1)$
b_1, a_2, a_3	$(-1, -1, -1)$
a_1, b_2, a_3	$(-1, -1, -1)$
a_1, a_2, b_3	$(-1, -1, -1)$
a_1, b_2, b_3	$(-1, 0, 0)$
b_1, a_2, b_3	$(-1, 0, 0)$
b_1, b_2, a_3	$(-1, 0, 0)$
b_1, b_2, b_3	$(-1, -1, -1)$

and $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. Consider the following Markov perfect strategy:

- at $-T$, player 1 takes a_1 and players 2 and 3 take b_2 and b_3, respectively,
- if (a_1, a_2, a_3) is prepared, no player changes the preparation,
- if (a_i, a_j, b_k) is prepared, player i prepares b_i, player j prepares b_j, and player k prepares a_k if they can move,
- if (a_i, b_j, b_k) is prepared, no player changes the preparation, and

[13] Such a player k does not exist for the two-player games. This is why (1) is automatically satisfied.

- if (b_1, b_2, b_3) is prepared, no player changes the preparation.

It is straightforward to verify this is an equilibrium and induces (a_1, b_2, b_3). It is, however, worth explaining why player i wants to prepare b_i instead of a_i in (a_i, a_j, b_k) at $-t$. Given that players follow the above strategy after $-t$, if player i prepares a_i, her expected payoff is

$$(1 - \exp(-3\lambda t)) \frac{1 + 0 + (-1)}{3} + \exp(-3\lambda t)(-1) < 0.$$

Let us explain the formula. With probability $(1 - \exp(-3\lambda t))$, some player can move. Conditional on this event, with probability $\frac{1}{3}$, the first mover will be player k, (a_1, a_2, a_3) will be induced, and player i will yield 1. With probability $\frac{1}{3}$, the first mover will be player i, (b_i, a_j, b_k) will be induced, and player i will yield 0. With probability $\frac{1}{3}$, the first mover will be player j, (a_i, b_j, b_k) will be induced, and player i will yield -1. Note that (a_1, a_2, a_3), (b_i, a_j, b_k), and (a_i, b_j, b_k) are absorbing states. With probability $\exp(-3\lambda t)$, no player can move and player i yields -1. On the other hand, taking b_i gives her 0. Therefore, it is optimal for player i to take b_i.

In the above example, even though it is common knowledge that player k will take a_k and induce the Pareto-dominant outcome (a_1, a_2, a_3) if she can move, if player j is the first mover, she will take b_j, which will hurt player i. Hence, it is optimal for player i to take b_i, which guarantees that she will get 0. By the same token, player j also fears that if player i is the first mover, it will hurt player j, which incentivizes her to take b_j.

An important assumption that is used to sustain this equilibrium is that, taking b_i (b_j, resp.) in (a_i, a_j, b_k) rescues player i (j, resp.) by giving 0 while it does not rescue player j (player i, resp.) at all. Consider player i's incentive to take b_i in (a_i, a_j, b_k). If player i takes a_i instead, then player j will take b_j and it will hurt player i. At the same time, player j wants to take b_j since player j expects that if player i moves afterwards, then player i will take b_i, which does not rescue player j. Condition (1) implies that when b_i (b_j, resp.) rescues player i (player j, resp.), it will also rescue player j (player i, resp.) at least slightly. If so, player i is willing to stay at (a_i, a_j, b_k) and wait for player k to induce the Pareto-dominant outcome

(a_1, a_2, a_3) regardless of player j's strategy. Hence, we can show that the Pareto-dominant outcome (a_1, a_2, a_3) is selected.

Finally, notice that the above example implies that Condition (1) is tight. In this game, we have

$$\begin{aligned} K &= 2 \\ r_i &= \frac{1}{3} \text{ for all } i. \end{aligned}$$

Hence,

$$\min_{i,j\in I, i\neq j}(r_i r_j) + \left(1 - \min_{i,j\in I, i\neq j}(r_i + r_j)\right)\left(1 - K\left(1 - \min_{i\in I} r_i\right)\right) = 0.$$

Notice that we can replace -1's in the above example by -1 $- l$ for any $l > 0$. This would make the left hand side of the above inequality strictly negative $(-\frac{2}{9}l)$.

5 Pareto-Unranked Nash Equilibria in a Component Game

The result in the previous section suggests that the strictly Pareto-dominant Nash equilibrium is selected if it exists. But what happens if there are multiple Pareto-optimal Nash equilibria? In this section, we answer this question for two-player games with two strict pure Pareto-unranked Nash equilibria. We show that in this class of games, there is a nongeneric set of parameters where the equilibrium payoff set is a full-dimensional subset of the feasible payoff set. On the other hand, for any parameter in the complement of the nongeneric set of parameters inducing multiple equilibria, there is a unique revision equilibrium.

Specifically, in this section, we consider a 2×2 game g with

	L	R
U	$\pi_1(U,L), \pi_2(U,L)$	$\pi_1(U,R), \pi_2(U,R)$
D	$\pi_1(U,D), \pi_2(U,D)$	$\pi_1(D,R), \pi_2(D,R)$

with two strict pure strategy equilibrium (U, L) and (D, R) and

$$\pi_1(U, L) > \pi_1(D, R), \tag{2}$$

$$\pi_2(U, L) < \pi_2(D, R). \tag{3}$$

(2) implies that player 1 prefers (U, L) to (D, R) among pure Nash equilibria while (3) implies that player 2's preference is opposite.

As we mentioned in the Introduction, the equilibrium selection depends on a joint condition of the payoff structure and the arrival rates. The key condition is whether

$$\begin{aligned} f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) \equiv\ & \frac{\lambda_2}{\lambda_1} \frac{\pi_2(U,L) - \pi_2(U,R)}{\pi_2(D,R) - \pi_2(U,L)} + \frac{\pi_2(D,R) - \pi_2(U,R)}{\pi_2(D,R) - \pi_2(U,L)} \\ & - \left\{ \frac{\lambda_1}{\lambda_2} \frac{\pi_1(D,R) - \pi_1(U,R)}{\pi_1(U,L) - \pi_1(D,R)} + \frac{\pi_1(U,L) - \pi_1(U,R)}{\pi_1(U,L) - \pi_1(D,R)} \right\} \end{aligned}$$

is zero, positive, or negative. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$, then the equilibrium payoff set is full dimensional. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) > 0$, then (U, L), the equilibrium preferable to player 1, is selected. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) < 0$, then (D, R), the equilibrium preferable to player 2, is selected.[14]

We postpone the intuitive explanation of the formula to Section 6.2. Let us formally state our result:

Proposition 6 *1. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$, then, the revision equilibrium payoff set $\phi^r(g)$ satisfies*

$$\phi^r(g) = \left\{ \begin{array}{c} a_1(\pi_1(U,L), \pi_2(U,L)) + a_2(\pi_1(D,R), \pi_2(D,R)) + a_3(\pi_1(D,R), \pi_2(U,L)) \\ \mid \sum_{k=1}^{3} a_k = 1, a_1, a_2, a_3 \geq 0 \text{ with } a_3 = 0 \Rightarrow (a_1, a_2) \in \{(1,0), (0,1)\} \end{array} \right\}.$$

2. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) > 0$, then (U, L) is a unique revision equilibrium.

3. If $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) < 0$, then (D, R) is a unique revision equilibrium.

[14] This condition coincides with the one already found by Calcagno and Lovo (2010), and did not appear in the earlier version of our paper because of an algebra error.

6 Intuitive Explanation

In this section, we explain the logic behind **Proposition 6**. Without loss of generality, we can concentrate on the case with $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) \geq 0$ since the other case is symmetric.

6.1 Three State Example

6.1.1 Setting

To gain some intuition for **Proposition 6**, this subsection considers a simplified setting of a revision game-like model. Suppose that there exist two players, 1 and 2, and three states, s_0, s_1 and s_2, in which payoffs are, $(0, 0)$, $(2 + \epsilon, 1)$ and $(1, 2)$, respectively. Assume $\epsilon \geq 0$ without loss of generality. Time is $-t \in [-T, 0]$ as before. Each player $i = 1, 2$ has two actions, I_i (Insist) and Y_i (Yield). The initial state (the state at $-t = -T$) is determined as follows. Players simultaneously take an action from $\{I_i, Y_i\}$. If player i takes I_i and player $j \neq i$ takes Y_j, s_i is realized at $-T$. If two players take (Y_1, Y_2) or (I_1, I_2), s_0 is realized at $-T$. When the current state is s_0 at $-t > -T$, each player i has a chance to take an action according to a Poisson process with arrival rate $\lambda_i > 0$, where the arrival rates satisfy $\frac{\lambda_1}{\lambda_2} = 1 + \delta$. If a Poisson process arrives when the state is s_0, players can choose an action from $\{I_i, Y_i\}$. Action I_i does not change the state, while action Y_i changes the state from s_0 to s_{-i}. Thus, i's action Y_i induces the state that the opponent $-i$ favors the most. We suppose (*only in this subsection*) that states s_1 and s_2 are "absorbing states." That is, when the current state is s_1 or s_2, no player can change the state (See Figure 1). The payoff is determined by the state where players are in at $-t = 0$.

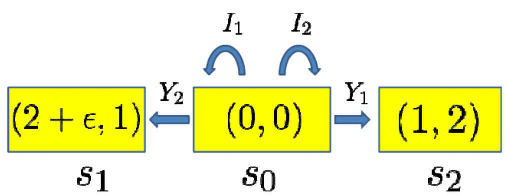

Figure 1: The transition of 3-state example

As in the analysis of the general model, we consider the limit case of $T \to \infty$, which is equivalent to the case when the frequency of revisions (λ_i) goes to infinity with T and δ fixed.

There are two distinct cases, $\epsilon = \delta$ and $\epsilon \neq \delta$, which correspond to the cases with $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$ and $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) \neq 0$, respectively. We consider the former case in 6.1.2 and the latter case in 6.1.3.

6.1.2 The Symmetric Case: $\epsilon = \delta$

When $\epsilon = \delta$, we have $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$. **Proposition 6** states that, in this case, the equilibrium payoffs in revision games is a full dimensional subset of the feasible payoff set. We illuminate the intuition behind this result by utilizing the three state model.

Claim 7 *If $\epsilon = \delta$, the limit of the subgame perfect equilibrium payoff set is*[15]

$$S = \left\{ \begin{array}{c} a_1(2+\epsilon, 1) + a_2(1,2) + a_3(1,1) \\ \mid \sum_{k=1}^{3} a_k = 1, a_1, a_2, a_3 \geq 0 \ \textit{with} \ a_3 = 0 \Rightarrow (a_1, a_2) = (1,0) \ \textit{or} \ (0,1) \end{array} \right\}.$$

[15] The asymmetry of a_1, a_2 and a_3 comes from the fact that for any finite horizon length T, with a strictly positive probability no one gets a move, so obtaining payoffs at, for example, exactly $(2+\epsilon, 1)$ is impossible.

We omit the formal proof of this result (because it is straightforward from the proof of **Proposition 6**), but offer the intuitive explanation of the logic behind this result.

To ease the notation, let us further assume $\epsilon = \delta = 0$, that is, $\lambda_1 = \lambda_2 \equiv \lambda > 0$. Analogous arguments would establish results for cases of $\epsilon = \delta \neq 0$.

First, it is intuitive that players should use the cutoff strategies: there exists t_i^* for each i such that if the state is still in s_0 at time $-t$, player i takes I_i for $-t < -t_i^*$ while she takes Y_i for $-t \geq -t_i$.[16]

An important observation here is that, since two players are perfectly symmetric by assumption ($\epsilon = \delta = 0$), two players' cutoffs are the same, that is, $t_1^* = t_2^* \equiv t^*$, and at this cutoff $-t^*$, players' incentives are such that:

- given that both players take Y_i afterwards, each player i strictly prefers Y_i to I_i after the cutoff;
- given that the opponent j will take I_j until the cutoff, player i is indifferent between Y_i and I_i from $-T$ until the cutoff; and
- given that the opponent j will take Y_j as soon as possible, player i strictly prefers I_i to Y_i before $-t^*$.

This cutoff t^* is characterized by

$$1 = (1 - \exp(-2\lambda t^*)) \frac{2+1}{2}. \tag{4}$$

Intuitively, the left hand side of (4) is the payoff of taking Y_i right now. The right hand side is the payoff of taking I_i. At $-t^*$, the probability that some player can move afterwards is given by $(1 - \exp(-2\lambda t^*))$. Conditional on some player being able to move, with probability $\frac{1}{2}$, it will be the opponent who will take Y_j, and player i will get 2 in this case. Otherwise, player i will take Y_i to get 1.

[16] The player is indifferent when $-t = -t_i^*$ but the probability that she obtains a revision opportunity at $-t_i^*$ is zero, so it does not affect the calculation of ex ante expected payoffs.

Now, given the above observation, we can construct equilibria that each yields an extremum point of $\text{co}\{(2,1),(1,1),(1,2)\}$. Point $(2,1)$ can be obtained in an equilibrium where (I_1, Y_2) is played at $-T$ and for $-t > -T$, if the current state is s_0, then player 2 takes Y_2 as soon as she can move while player 1 takes I_1 before $-t^*$ and then takes Y_1 if she can move at or after $-t^*$. The above observation verifies that this is an equilibrium. Then, this equilibrium yields s_1. Point $(1,2)$ can be obtained by a symmetric manner. To obtain payoff profile $(1,1)$, we construct an equilibrium as follows: Players take (I_1, I_2) at $-T$ and when $-t < -t^*$, each player i sticks to I_i and when $-t \geq -t^*$, each player i takes Y_i. We refer to this equilibrium as a "chicken race equilibrium," as the equilibrium has a flavor of the "chicken race game," in which two drivers drive their cars towards each other until one of them gives in, while if both do not give in then the cars crash and the drivers die.

Given above, we can construct an equilibrium strategy which induces the payoff profile $p(2,1) + (1-p)(1,1)$ with $p \in [0,1]$: Player 1 takes I_1 at $-T$. Player 2 takes Y_2 with probability p and I_2 with probability $1-p$ at $-T$. If s_0 is realized, players play the "chicken race equilibrium." By definition of t^*, this is an equilibrium and induces $(2,1)$ if s_1 is realized at $-T$ and $(1,1)$ if s_0 is realized. Symmetrically, we can construct an equilibrium to attain $p(1,2) + (1-p)(1,1)$. with $p \in [0,1]$

Finally, we explain how to attain the payoffs other than the extreme points. Because we are not assuming any public randomization device, it is not obvious that these payoffs can be achieved. However, we can use the Poisson arrivals before some $-\bar{t}$ with $\bar{t}$ being sufficiently large as a public randomization device: at $-T$, players take I_i. After that, if s_0 is realized, then each player i takes I_i until $-\bar{t}$ and "count" the numbers of Poisson arrivals to each player until $-\bar{t}$ and then decide which of three equilibria to play after $-\bar{t}$:

- At $-T$, (I_1, I_2) is taken.
- For $-t < -\bar{t}$, each player i takes I_i until $-\bar{t}$.
- Depending on the realization of the Poisson process until $-\bar{t}$, players coordinate on either one of the following three equilibria:

– For $-t \in [-\bar{t}, -t^*)$, player 1 takes I_1 while player 2 takes Y_2. For $-t \in [-t^*, 0]$, each player takes Y_i.

– For $-t \in [-\bar{t}, -t^*)$, player 2 takes I_2 while player 1 takes Y_1. For $-t \in [-t^*, 0]$, each player takes Y_i.

– For $-t < -\bar{t}$, each player i takes I_i until $-\bar{t}$. For $-t \in [-\bar{t}, -t^*)$, each player i takes I_i. For $-t \in [-t^*, 0]$, each player takes Y_i.

If the length $[-T, -\bar{t}]$ is sufficiently large and $\bar{t}$ is sufficiently large, we can achieve any convex combination of $\{(2+\epsilon, 1), (1,1), (1,2)\}$, by appropriately specifying the length of time intervals on which players "count" the number of Poisson arrivals.

Hence, we have shown that **Claim 7** holds. As a summary, Figure 2 depicts the revision equilibrium payoff set with $\epsilon = \delta = 0$.

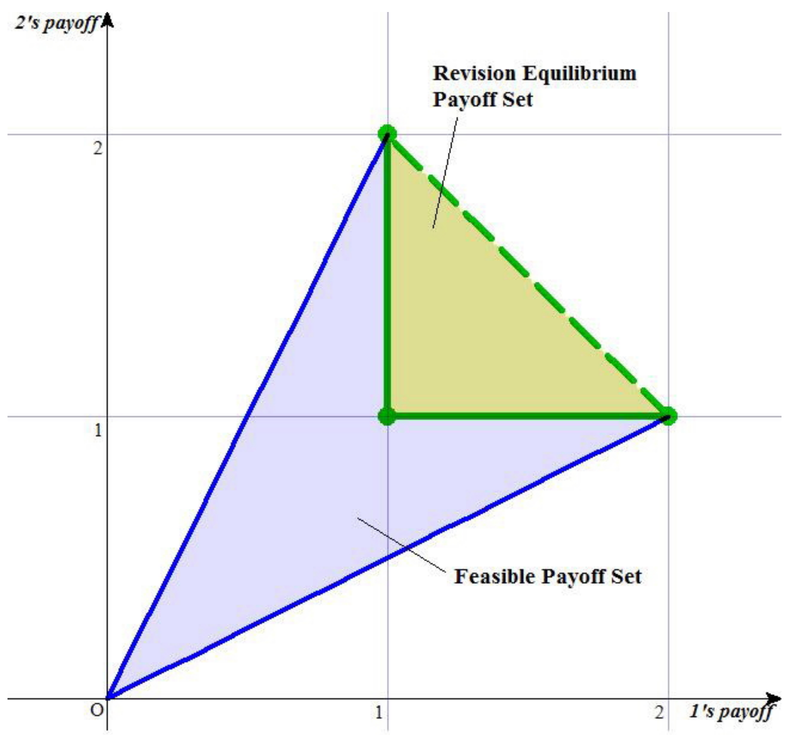

Figure 2: Symmetric case

6.1.3 The Asymmetric Case: $\epsilon \neq \delta$

The key property behind the existence of multiple equilibria (especially behind the "chicken race equilibrium") in the previous part is that $-t_i^*$, the time after which player i strictly prefers Y_i conditional on each player choosing Y_j afterwards, is the same across players. If $\epsilon \neq \delta$, however, this timing differs across players. Moreover, we can show that this asymmetry breaks down the "chicken race equilibrium," which gives us the uniqueness of equilibria.

Notice that $\epsilon \neq \delta$ corresponds to the case of $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) \neq 0$ in **Proposition 6**. We have an analogous result to **Proposition 6** as follows:

Claim 8 *If $\epsilon \neq \delta$, there is a unique subgame perfect equilibrium: If $\epsilon > \delta$, player* 2 *yields whenever she can move, and if $\epsilon < \delta$, player* 1 *yields whenever she can move.*

Again, we omit the proof of this claim, but will illuminate the intuition behind this result.

Without loss of generality, we assume $\epsilon > \delta$ since the other case is perfectly symmetric. To simplify the notation, let us further specialize to the case of $\delta = 0$, that is, $\lambda_1 = \lambda_2 \equiv \lambda$. Analogous arguments would establish results for cases of $\epsilon \neq \delta \neq 0$.

Consider player i's "critical point" $-t_i^{**}$ such that conditional on both players choosing Y_i afterwards,

- given that both players take Y_i afterwards, each player i strictly prefers Y_i to I_i after the critical point,
- given that the opponent j will take I_j until the critical point, player i is indifferent between Y_i and I_i from $-T$ to the critical point, and
- given that the opponent j will take Y_j as soon as possible, player i strictly prefers I_i to Y_i before $-t^{**}$.

The critical point for player 1, $-t_1^{**}$, is characterized by

$$1 = \left(1 - \exp\left(-2\lambda t_1^{**}\right)\right) \frac{(2+\epsilon)+1}{2}$$

as before. On the other hand, the critical point for player 2 is characterized by

$$1 = (1 - \exp(-2\lambda t_2^{**})) \frac{2+1}{2}.$$

Notice that $-t_2^{**} < -t_1^{**}$ holds. Actually this inequality holds more generally whenever $\epsilon > \delta$. This is intuitive: ϵ represents how strongly player 1 prefers s_2 to s_1. Thus it measures the willingness of player to stick with s_0. On the other hand, δ represents how strong player 2's commitment power is compared to player 1's commitment power and so it represents how strongly player 1 is forced to yield. $\epsilon > \delta$ implies that the former effect is stronger than the latter effect.

This implies that the possible cutoff strategy profile is $-t_1^* = -t_1^{**}$ and $-t_2^* = -T$, that is, player 1 takes I_1 until $-t_1^{**}$ while player 2 takes Y_2 as soon as possible. Let us explain why this is a unique equilibrium, in the following two steps.

First, we show that $-t_2^* < -t_1^*$. Suppose, to the contrary, that $-t_1^* \leq -t_2^*$. At $-t_2^*$, we can suppose that each player i will take Y_i afterwards. At $-t_2^{**}$, player 2 should be indifferent between Y_2 and I_2, that is, $-t_2^* = -t_2^{**}$. The fact that $-t_2^{**} < -t_1^{**}$ implies that player 1 strictly prefers I_1 to Y_1 at $-t_2^*$, which contradicts our starting assumption that $-t_1^* \leq -t_2^*$. Hence, we have $-t_2^* < -t_1^*$.

Second, given $-t_2^* < -t_1^*$, we can show $-t_1^* = -t_1^{**}$. At $-t_1^*$, we can condition that each player i will take Y_i afterwards. In addition, at $-t_1^*$, player 1 should be indifferent between Y_1 and I_1. By definition, therefore, $-t_1^* = -t_1^{**}$.

Third, we show $-t_2^* = -T$. Notice that $-t_1^* = -t_1^{**}$. Player 1 keeps taking I_1 at least until $-t_1^{**}$. $-t_2^{**} < -t_1^{**}$ implies that if they are in s_0 at $-t_1^{**}$, player 2's expected payoff is strictly less than 1. Suppose that player 2 obtains a revision opportunity at $-t < -t_1^* = -t_1^{**}$. Taking Y_2 gives her the payoff of 1, while taking I_2 gives her a payoff strictly less than 1. This is because all what can happen before $-t_1^* = -t_1^{**}$ is to stay at s_0 or to go to s_1, so player 2's expected payoff of taking I_2 is a convex combination of the payoff from staying s_0 at $-t_1^* = t_1^{**}$ (strictly less than 1) and that from being s_1 at $-t_1^* = t_1^{**}$ (equal to 1). Since the former happens with a strictly positive probability (it is possible that no player gets a revision opportunity until $-t_1^*$), player 2's expected payoff of taking I_2 is strictly less than 1. This means that player 2 should take Y_2 whenever possible.

Hence, we have shown that **Claim 8** holds. As a summary, Figure 3 depicts the revision equilibrium payoff set with $\epsilon > \delta$.

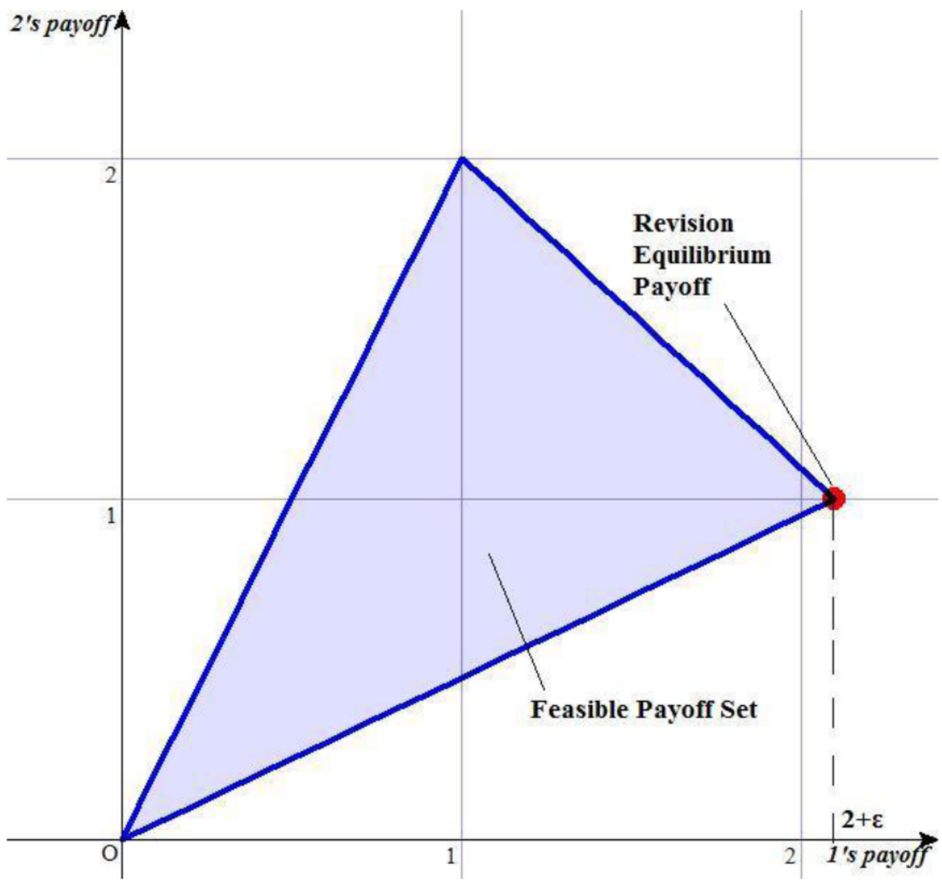

Figure 3: Unique equilibrium

6.2 Equilibrium Dynamics

Given the example above, let us interpret $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right)$ intuitively. If $\lambda_1 = \lambda_2$, $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right)$ is simplified as

$$f^*(\pi, 1) = 2\left\{\frac{\pi_2(D,R) - \pi_2(U,R)}{\pi_2(D,R) - \pi_2(U,L)} - \frac{\pi_1(U,L) - \pi_1(U,R)}{\pi_1(U,L) - \pi_1(D,R)}\right\}.$$

This formula compares how strongly each player likes the Nash equilibrium preferable to herself relative to the starting point of the "chicken race," where players insist a non-equilibrium action profile (U, R), rather than giving into the Nash equilibrium preferable to the opponent. In particular, if $\pi_1(U, L)$ increases, then $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right)$ increases. That is, if player 1 prefers (U, L), her preferable Nash equilibrium, more, then player 1 becomes "stronger" and (U, L) is more likely to be selected. In addition, since $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right)$ is decreasing in $\frac{\lambda_1}{\lambda_2}$, if player 2's relative frequency compared to player 1's frequency decreases, then player 2's commitment power becomes stronger and it hurts player 1, that is, (U, L) is less likely to be selected.[17]

The above example and intuitive explanation capture an essential argument for the case with $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) \neq 0$: a slight payoff asymmetry leads to a slight difference of the incentives, which causes a huge difference in the equilibrium behavior in the dynamic setting. We note, however, that the analysis of the general setting is substantially more difficult than the example we discussed here especially in the case with a unique equilibrium. The reason is that neither s_1 nor s_2 is a priori absorbing states since players can revise their actions in s_1 and s_2. Indeed, it will turn out that the action profile that corresponds to state s_2 is not absorbing in the general model.

Actual equilibrium dynamics in a general 2×2 games with $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) > 0$ are illustrated

[17] Note that our equilibrium selection can select the risk dominated equilibrium. Consider a payoff matrix

$$\begin{array}{ccc} & L & R \\ U & 2+\epsilon, 1 & 0, 0 \\ D & 2\epsilon, 0 & 1, 2 \end{array}.$$

Even though (U, L) is risk dominated by (D, R), (U, L) is selected by the revision game.

by the following figures. If $-t$ is sufficiently close to 0, every player prepares the static best response to the other as illustrated in Figure 4.

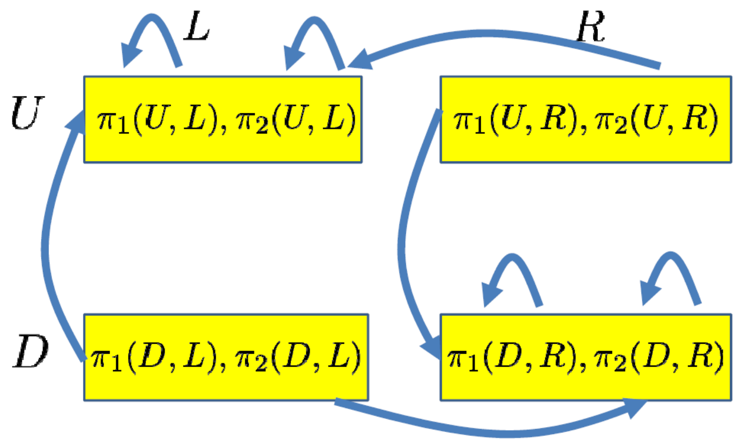

Figure 4: Dynamics when $-t$ is close to 0

Then, suppose (U, R) is prepared at time $-t$. As we briefly mentioned in Section 6.1.3, the "strong" player 1 sticks to (U, R) if $-t$ is sufficiently far away from 0, knowing that the “weak” player 2 gives in and goes to (U, L) if she can move afterwards. See Figure 5.

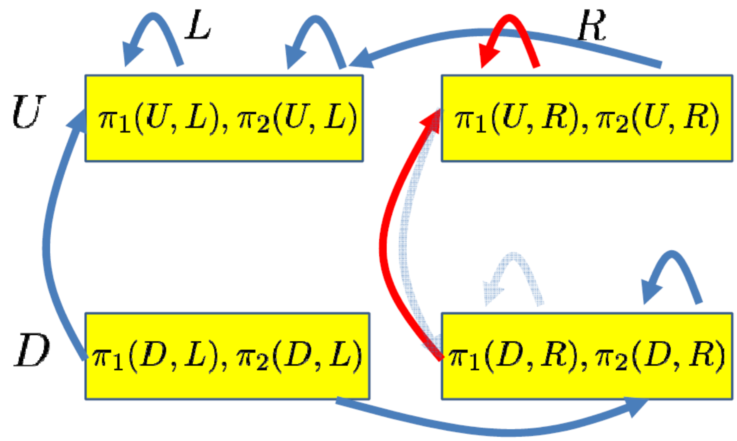

Figure 5: Dynamics when $-t$ is in a middle range

There is another complication that arises when $-t$ is further away from the deadline, 0. Now, player 2 prepares L when player 1 prepares D. Thus, players prepare the revision equilibrium action profile, (U, L), as soon as possible. By doing so, they maximize the probability to obtain the payoffs in (U, L).

It turns out that this transition rule remains the same for all larger t's. This is why (U, L) is the revision equilibrium, the unique element in the revision equilibrium set.

The transition rule is summarized in Figure 6.

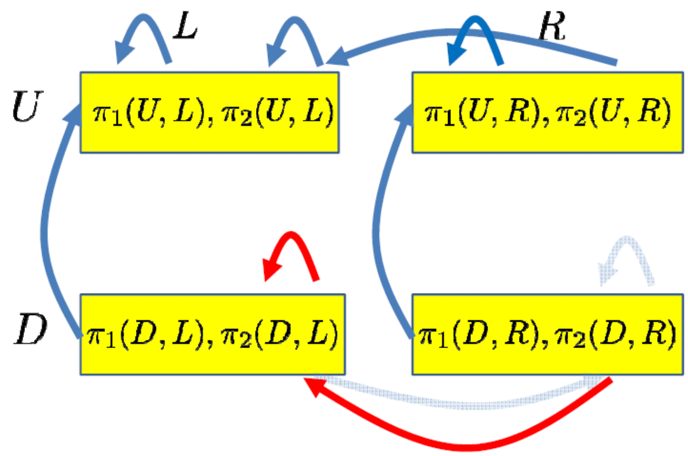

Figure 6: Dynamics when $-t$ is far away from 0

7 Concluding Remarks

We analyzed the situation in which two players prepare their actions before they play a normal-form coordination game at a predetermined date. In the preparation stage, each player stochastically obtains opportunities to revise their actions, and the finally-revised action is played at the deadline. We show that, (i) if there exists a Nash equilibrium that strictly Pareto-dominates all the other action profiles, then it is the only equilibrium if the component game is sufficiently close to a pure coordination game; and (ii) in 2×2 games with two strict Pareto-unranked pure Nash equilibria, (ii-a) while with perfectly symmetric structure, the equilibrium set is a full-dimensional subset of the feasible payoff set, (ii-b) a slight asymmetry is enough to select a unique equilibrium, which corresponds to the Nash equilibrium in the static game that gives the highest payoff to the "strong" player.

Let us mention possible directions of future research. First, our analysis has been restricted to games with a strictly Pareto-dominant Nash equilibrium that are close to a pure coordination games or 2×2 games with two strict pure Nash equilibrium, but the basic intuition seems to extend to more general cases. Second, even in a 2×2 game, we have not yet considered all possible cases. For example, in the following game, the outcome of the revision game is not straightforward:

	L	R
U	$10, 11$	$-1, 10$
D	$11, -1$	$0, 0$

In this game, D is a dominant action for player 1, while player 2's best response depends on player 1's action. Although (D, R) is the unique Nash equilibrium, its payoff is Pareto-dominated by that of (U, L). Hence, it is not obvious whether players want to switch to the Nash action as soon as possible.

Third, it would be interesting to consider the case in which there exists only one mixed equilibrium. For example, in a symmetric "matching pennies" game, it is obvious that the probability distribution over the outcome is the same as in the mixed strategy equilibrium of the component game. A question is whether this holds true for the asymmetric case.

Fourth, it would be interesting to see the hybrid version of synchronized and asynchronized revision games.[18]

These possibilities are beyond the scope of this paper, but we believe that the present paper provides an important first step towards these generalizations.

8 Appendix: Proof of Propositions

Since fixing T and $(r_1, \ldots, r_n)$ and letting $(\lambda_1, \ldots, \lambda_n)$ converge to infinity is equivalent to fixing $(\lambda_1, \ldots, \lambda_n)$ and letting T converge to infinity, we consider the latter formulation for the sake of "backward induction."

The following notations are useful: let σ_i be a strategy in the revision game. Generally, σ_i is a mapping from a history at $-t$, h_t, to a distribution over preparation ΔX_i. As we will see, in many cases, σ_i is pure and Markov perfect, that is, it only depends on the calendar

[18] Calcagno and Lovo (2010) investigates this possibility deeper than this paper.

time t and the action profile most recently prepared by the players.[19] Therefore, letting x_i be the action most recently prepared by player i at t, we can write $\sigma_i^t(x) \in X_i$. Finally, let $BR_i(x_{-i})$ be a static best response to x_{-i} by player i in the component game.

8.1 Proof of Proposition 4

Firstly, we prove the following two lemmas:

Lemma 9 *Suppose that x^* is strictly Pareto-dominant Nash equilibrium strategy profile. Then if a subgame starts with a profile x^*, there is a unique subgame perfect equilibrium in that subgame and this equilibrium designates actions x_i^* for each i on the equilibrium path of the play.*

Proof. Since x^* is strictly Pareto-dominant and X is finite, there exists $\epsilon > 0$ such that for all i and all $x \in X$ with $x \neq x^*$, $\pi_i(x_i^*) > \pi_i(x) + \epsilon$.

The lower bound of the payoff from taking action x_i^* at time $-\delta$ given the opponent's current action x_{-i}^* is

$$\exp\left(-\delta\left(\textstyle\sum_{j\in I}\lambda_j\right)\right)\pi_i(x^*) + \left(1-\exp\left(-\delta\left(\textstyle\sum_{j\in I}\lambda_j\right)\right)\right)\underline{\pi}_i$$

with

$$\underline{\pi}_i = \min_{x\in X}\pi_i\left(x\right).$$

The upper bound of the payoff from taking action $\hat{x}_i \neq x_i^*$ at time $\delta > 0$ given the opponent's current action x_{-i}^* is

$$\exp\left(-\delta\left(\textstyle\sum_{j\in I}\lambda_j\right)\right)\pi_i(\hat{x}_i, x_{-i}^*) + \left(1-\exp\left(-\delta\left(\textstyle\sum_{j\in I}\lambda_j\right)\right)\right)\pi_i(x^*).$$

[19] Since the probability that two players can move simultaneously is 0, we ignore this probability.

Hence taking x_i^* is strictly better at time $-\delta$ conditional on any history if

$$
\begin{aligned}
& \exp\left(-\delta\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(x^*) + \left(1-\exp\left(-\delta\left(\sum_{j\in I}\lambda_j\right)\right)\right)\underline{\pi} \\
> \; & \exp\left(-\delta\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(\hat{x}_i, x_{-i}^*) + \left(1-\exp\left(-\delta\left(\sum_{j\in I}\lambda_j\right)\right)\right)\pi_i(x^*)
\end{aligned}
$$

$\Leftrightarrow$

$$
0 \le \delta < \frac{1}{\sum_{j\in I}\lambda_j}\ln\left(\frac{\epsilon}{\pi_i(x^*)-\underline{\pi}_i}+1\right) \in \mathbb{R}_{++}.
$$

Let

$$
\delta^* = \min_{i\in I}\left\{\frac{1}{\sum_{j\in I}\lambda_j}\ln\left(\frac{\epsilon}{\pi(x^*)-\underline{\pi}_i}+1\right)\right\} > 0.
$$

Then, at any $-t \in (-t^*, 0]$, each player i chooses x_i^* at t conditional on the opponent's current action x_{-i}^*.

Now we make a backward induction argument. Suppose that for all time $-t \in (-k\delta^*, 0]$, each player i chooses x_i^* conditional on the opponent's current action x_{-i}^*. We show that at any $-t' \in (-(k+1)\delta^*), -k\delta^*]$, each player i chooses x_i^* conditional on the opponent's current action x_{-i}^*.

The lower bound of the payoff from taking action x_i^* at time $-t \in (-(k\delta^*+\delta'), -k\delta^*]$ with $\delta' > 0$ is

$$
\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(x^*) + \left(1-\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\right)\underline{\pi}_i.
$$

The upper bound of the payoff from taking action $\hat{x}_i \neq x_i^*$ is

$$
\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(\hat{x}_i, x_{-i}^*) + \left(1-\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\right)\pi_i(x^*).
$$

Hence taking x_i^* is strictly better at time δ' conditional on the opponent's current action x_{-i}^* if

$$\begin{aligned} & \exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(x^*)+\left(1-\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\right)\underline{\pi}_i \\ > \; & \exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\pi_i(\hat{x}_i,x_{-i}^*)+\left(1-\exp\left(-\delta'\left(\sum_{j\in I}\lambda_j\right)\right)\right)\pi_i(x^*) \end{aligned}$$

$\Leftrightarrow$

$$\delta' < \frac{1}{\sum_{j\in I}\lambda_j}\ln\left(\frac{\epsilon}{\pi(x^*)-\underline{\pi}_i}+1\right).$$

Thus each player i strictly prefers playing x_i^* at all time $-t' > -k\delta^* - \min_{i\in I}\left\{\frac{1}{\sum_{j\in I}\lambda_j}\ln\left(\frac{\epsilon}{\pi(x^*)-\underline{\pi}_i}+1\right)\right\} = (k+1)\delta^*$ conditional on the opponent's current action x_{-i}^*. This completes the proof. ■

Now we prove the proposition by mathematical induction. Let $\underline{V}_m^i(t)$ be the infimum of player i's payoff at $-t$ with respect to the subgame perfect strategies and histories such that at least m players take x_j^* at $-t$. We will show, by mathematical induction with respect to m, that $\underline{V}_m^i(t)$ is sufficiently close to $\pi_i(x^*)$ when t is sufficiently large.

By **Lemma 9**, when $m=n$, $\underline{V}_m^i(t)=\pi_i(x^*)$ for all i,m.

Suppose $\lim_{t\to\infty}\underline{V}_{m+1}^i(t)=\pi_i(x^*)$. We want to show $\lim_{t\to\infty}\underline{V}_m^i(t)=\pi_i(x^*)$. For simple notation, let us denote

$$\begin{aligned} \alpha_1 &= \min_j r_j, \\ \beta &= \min_{j\in I\setminus\{j^*\}} r_j \text{ with } j^*\in\arg\min_j r_j, \\ \bar{K} &= \max_{i,j}\frac{\pi_i(x^*)-\underline{\pi}_i}{\pi_j(x^*)-\underline{\pi}_j}\geq K. \end{aligned}$$

Take $\xi>0$ arbitrarily. Since $\lim_{t\to\infty}\underline{V}_{m+1}^i(t)=\pi_i(x^*)$, there exists $T_0(\xi)$ such that

$$\underline{V}_{m+1}^i(t)\geq\pi_i(x^*)-\xi$$

for all $t\geq T_0(\xi)$. Consider the situation where m players take x_j^* at $-t=-(T_0(\xi)+\tau)$. Then, if player j that is not taking x_j^* at time $-t$ will move next by $-T_0(\xi)$, then player

j will yield at least $\pi_j(x^*) - \xi$ by taking x_j^*. This implies each player i will at least yield $\pi_i(x^*) - \bar{K}\xi$ in this case. Therefore,

$$\begin{aligned}\underline{V}_m^i(t) &\geq \alpha_1\left(1 - \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right)\left(\pi_i(x^*) - \bar{K}\xi\right) \\ &\quad + \left(1 - \alpha_1\left(1 - \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right)\right)\underline{\pi}_i\end{aligned}$$

for all i. Note that there exists $T_1(\xi)$ such that for all $\tau \geq T_1(\xi)$,

$$\underline{V}_m^i(t) \geq \alpha_1\pi_i(x^*) + (1-\alpha_1)\underline{\pi}_i - \bar{K}\xi$$

for all i.

Consider $\underline{V}_m^i(t)$ with $t = T_0(\xi) + T_1(\xi) + \tau$. Then, player j's payoff is at least

- $\pi_i(x^*) - \bar{K}\xi$ if player j that is not taking x_j^* at time $-t$ will move first by $-(T_0(\xi) + T_1(\xi))$,
- $\alpha\pi_i(x^*) + (1-\alpha_1)\underline{\pi}_i - \bar{K}\xi$ if player i herself will move first or no player can move by $-(T_0(\xi) + T_1(\xi))$, and
- $(1 - K(1-\alpha_1))\pi_i(x^*) + K(1-\alpha_1)\underline{\pi}_i - \bar{K}^2\xi$ if player j that is taking x_j^* at $-t$ will move first by $-(T_0(\xi) + T_1(\xi))$. Note that, in this case, player j's value is bounded from below by $\alpha\pi_j(x^*) + (1-\alpha_1)\underline{\pi}_j - \bar{K}\xi$. By definition of K, this implies player i's value is bounded by $(1 - K(1-\alpha_1))\pi_i(x^*) + K(1-\alpha_1)\underline{\pi}_i - \bar{K}^2\xi$.

Therefore, player i's value satisfies

$$\begin{aligned}\underline{V}_m^i(t) &\geq \alpha_1\left(1 - \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right)\left(\pi_i(x^*) - \bar{K}\xi\right) \\ &\quad + \left(\beta\left(1 - \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right) + \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right)\left(\alpha_1\pi_i(x^*) + (1-\alpha_1)\underline{\pi}_i - \bar{K}\xi\right) \\ &\quad + (1-\alpha_1-\beta)\left(1 - \exp\left(-\tau \textstyle\sum_{j\in I}\lambda_j\right)\right)\left((1 - K(1-\alpha_1))\pi_i(x^*) + K(1-\alpha_1)\underline{\pi}_i - \bar{K}^2\xi\right).\end{aligned}$$

Note that there exists $T_2(\xi)$ such that for all $\tau \geq T_2(\xi)$,

$$\begin{aligned}
\underline{V}_m^i(t) &\geq (\alpha_1 + \alpha_1\beta + (1-\alpha_1-\beta)(1-K(1-\alpha_1)))\,\pi_i(x^*) \\
&\quad + (1-(\alpha_1+\alpha_1\beta+(1-\alpha_1-\beta)(1-K(1-\alpha_1))))\,\underline{\pi}_i \\
&\quad -\bar{K}^2\xi \\
&= \alpha_2\pi_i(x^*) + (1-\alpha_2)\,\underline{\pi}_i - \bar{K}^2\xi
\end{aligned}$$

with

$$\alpha_2 \equiv (\alpha_1 + \alpha_1\beta + (1-\alpha_1-\beta)(1-K(1-\alpha_1))).$$

Recursively, if (1) is satisfied, there exists M and $\{T_m(\xi)\}_{m=1}^M$ with M being independent of ξ such that for all $t \geq \sum_{m=0}^M T_m(\xi)$,

$$\underline{V}_m^i(t) \geq \alpha_M\pi_i(x^*) + (1-\alpha_M)\,\pi_i - \bar{K}^M\xi$$

with

$$\alpha_M \geq 1.$$

Taking ξ going to 0 yields the result.

8.2 Proof of Proposition 6

8.2.1 Case I: $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$

We show that if $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) = 0$, then, the revision equilibrium payoff set $\phi^r(g)$ satisfies

$$\phi^r(g) = \left\{ \begin{array}{c} a_1(\pi_1(U,L), \pi_2(U,L)) + a_2(\pi_1(D,R), \pi_2(D,R)) + a_3(\pi_1(D,R), \pi_2(U,L)) \\ \mid \sum_{k=1}^3 a_k = 1, a_1, a_2, a_3 \geq 0 \text{ with } a_3 = 0 \Rightarrow (a_1, a_2) \in \{(1,0),(0,1)\} \end{array} \right\}.$$

First, we prove the existence of an equilibrium with expected payoff $(\pi_1(D,R), \pi_1(U,L))$

by construction: let t^* be

$$t^* = \frac{1}{\lambda_1+\lambda_2}\log\left(\frac{\lambda_1}{\lambda_2}\frac{\pi_1(D,R)-\pi_1(U,R)}{\pi_1(U,L)-\pi_1(D,R)}+\frac{\pi_1(U,L)-\pi_1(U,R)}{\pi_1(U,L)-\pi_1(D,R)}\right) \tag{5}$$
$$= \frac{1}{\lambda_1+\lambda_2}\log\left(\frac{\lambda_2}{\lambda_1}\frac{\pi_2(U,L)-\pi_2(U,R)}{\pi_2(D,R)-\pi_2(U,L)}+\frac{\pi_2(D,R)-\pi_2(U,R)}{\pi_2(D,R)-\pi_2(U,L)}\right).$$

Note that at t^*, staying at (U,R) gives $(\pi_1(D,R),\pi_1(U,L))$ to both players, given that for $-t \in (-t^*,0]$, $\sigma_i^t(x) = BR_i(x)$.

Second, we prove the existence of an equilibrium with expected payoff

$$p(\pi_1(U,L),\pi_2(U,L)) + (1-p)(\pi_1(D,R),\pi_2(U,L))$$

with $p \in [0,1]$:

- Player 1 takes U at $-T$.
- Player 2 takes L with probability p and R with probability $1-p$ at $-T$.
- Player 1 takes the following Markov strategy:
  - For $-t \in [-T, t^*)$, $\sigma_1^t(x) = U$ for all x.
  - For $-t \in [-t^*, 0]$, $\sigma_1^t(x) = BR_1(x)$.
- Player 2 takes the following Markov strategy:
  - For $-t \in [-T, t^*)$, $\sigma_2^t(U,L) = L$ and $\sigma_2^t(U,R) = \sigma_2^t(D,L) = \sigma_2^t(D,R) = R$.
  - For $-t \in [-t^*, 0]$, $\sigma_2^t(x) = BR_2(x)$.

Then, (5) gives the condition that, given that players takes $BR_i(x)$ after $-t^*$, player 1 is indifferent between U and D at $x = (U,R),(D,R)$ and player 2 is indifferent between R and L at $x = (U,R),(U,L)$.

Given this, at (U,R) and (D,R) at $-t^*$, the expected payoff for player 1 is $\pi_1(D,R)$. If (U,R) or (D,R) is the state at $-t < -t^*$, then the above strategy profile implies that the state at $-t^*$ should be either (U, R) or (D, R). Therefore, at (U, R) and (D, R) at $-t \leq -t^*$, the expected payoff for player 1 is $\pi_1(D, R)$ regardless of player 1's strategy at $-t$. If (U, L) or (D, L) is the state, U is strictly optimal. The symmetric argument verifies the optimality of player 2's strategy.

Third, symmetrically, there exists an equilibrium with expected payoff

$$p\left(\pi_1(D, R), \pi_2(D, R)\right) + (1-p)\left(\pi_1(D, R), \pi_2(U, L)\right)$$

with $p \in [0, 1]$:

- Player 1 takes D with probability p and U with probability $1 - p$ at $-T$.
- Player 2 takes R at $-T$.
- Player 1 takes the following Markov strategy:
  - For $-t \in [-T, , t^*)$, $\sigma_1^t(D, R) = D$ and $\sigma_1^t(U, R) = \sigma_1^t(U, L) = \sigma_1^t(D, L) = U$.
  - For $-t \in [-t^*, 0]$, $\sigma_1^t(x) = BR_1(x)$.
- Player 2 takes the following Markov strategy:
  - For $-t \in [-T, t^*)$, $\sigma_2^t(x) = R$ for all x.
  - For $-t \in [-t^*, 0]$, $\sigma_2^t(x) = BR_2(x)$.

Fourth, we prove the existence of an equilibrium with expected payoff close to $\left(\pi_1(U, L), \pi_2(U, L)\right)$ by construction. Let $\bar{T} > 0$ be a large number. We verify that the following strategy profile constitutes an equilibrium:

- Player 1 takes U at $-T$.
- Player 2 takes R at $-T$.
- Player 1 takes the following Markov strategy:

- for $-t \in [-T, t^*)$, $\sigma_1^t(x) = U$ for all x.
- for $-t \in [-t^*, 0]$, $\sigma_1^t(x) = BR_1(x)$.

- Player 2 takes the following Markov strategy:
  - for $-t \in [-T, -\bar{T})$, $\sigma_2^t(x) = L$ for all x.
  - for $-t \in [-\bar{T}, 0]$, $\sigma_2^t(x) = BR_2(x)$.

Note that for sufficiently large $\bar{T}$, the expected payoff is sufficiently close to $(\pi_1(U, L), \pi_2(U, L))$. The same argument as before verifies that the above strategy profile constitutes an equilibrium.

Symmetrically, the following equilibrium approximates $(\pi_1(D, R), \pi_2(D, R))$.

- Player 1 takes U at $-T$.
- Player 2 takes R at $-T$.
- Player 1 takes the following Markov strategy:
  - For $-t \in [-T, -\bar{T})$, $\sigma_1^t(x) = U$ for all x.
  - For $-t \in [-\bar{T}, 0]$, $\sigma_1^t(x) = BR_1(x)$.
- Player 2 takes the following Markov strategy:
  - For $-t \in [-T, -t^*)$, $\sigma_2^t(x) = R$ for all x.
  - For $-t \in [-t^*, 0]$, $\sigma_2^t(x) = BR_2(x)$.

Therefore, we construct equilibria approximating $(\pi_1(D, R), \pi_2(U, L))$, $(\pi_1(U, L), \pi_2(U, L))$, and $(\pi_1(D, R), \pi_2(D, R))$. Note that in these three equilibrium, nobody moves until $-\bar{T}$ and the following is also an equilibrium for any N, M: with $T^* > \bar{T}$,

- Player 1 takes U at $-T$.

- Player 2 takes R at $-T$.
- Player 1 takes the following Markov strategy:
  - For $-t \in [-T, -T^*)$, $\sigma_1^t(x) = U$ for all x.
  - If the number of chances where player 1 can move for $[-T, -T^*)$ is less than N, going to the first equilibrium, that is,
    * For $-t \in [-T^*, -t^*)$, $\sigma_1^t(x) = U$ for all x.
    * For $-t \in [-t^*, 0]$, $\sigma_1^t(x) = BR_1(x)$.
  - If the number of chances where player 1 can move for $[-T, -T^*]$ is no less than N and no more than M, going to the second equilibrium, that is,
    * For $-t \in [-T^*, -t^*)$, $\sigma_1^t(x) = U$ for all x.
    * For $-t \in [t^*, 0]$, $\sigma_1^t(x) = BR_1(x)$.
  - If the number of chances where player 1 can move for $[-T, -T^*]$ is no more than M, going to the third equilibrium, that is,
    * For $-t \in [-T^*, -\bar{T})$, $\sigma_1^t(U, R) = U$ for all x.
    * For $-t \in [-\bar{T}, 0]$, $\sigma_1^t(x) = BR_1(x)$.
- Player 2 takes the following Markov strategy:
  - For $-t \in [-T, -T^*)$, $\sigma_2^t(U, L) = R$ for all x.
  - If the number of chances where player 1 can move for $[-T, -T^*]$ is less than N, going to the first equilibrium, that is,
    * For $-t \in [-T^*, -t^*)$, $\sigma_2^t(x) = R$ for all x.
    * For $-t \in [-t^*, 0]$, $\sigma_2^t(x) = BR_2(x)$.
  - If the number of chances where player 1 can move for $[-T, -T^*]$ is no less than N and no more than M, going to the second equilibrium, that is,

* For $-t \in [-T^*, \bar{T})$, $\sigma_2^t(x) = R$ for all x.
* For $-t \in [-\bar{T}, 0]$, $\sigma_2^t(x) = BR_2(x)$

– If the number of chances where player 1 can move for $[-T, -T^*]$ is no more than M, going to the third equilibrium, that is,

* For $-t \in [-T^*, -t^*)$, $\sigma_2^t(x) = R$ for all x.
* for $-t \in [-t^*, 0]$, $\sigma_2^t(x) = BR_2(x)$.

It is straightforward to show that this is an equilibrium and with appropriate choices of T^*, N, M, and sufficiently large $\left|T^* - \bar{T}\right|$, we can attain any payoff profile that can be expressed by a convex combination of $(\pi_1(D,R), \pi_2(U,L))$, $(\pi_1(U,L), \pi_2(U,L))$, and $(\pi_1(U,L), \pi_2(U,L))$.

Hence, we have shown that

$$\phi^r(g) \supseteq \left\{ \begin{array}{c} a_1(\pi_1(U,L), \pi_2(U,L)) + a_2(\pi_1(D,R), \pi_2(D,R)) + a_3(\pi_1(D,R), \pi_2(U,L)) \\ \mid \sum_{k=1}^{3} a_k = 1, a_1, a_2, a_3 \geq 0 \text{ with } a_3 = 0 \Rightarrow (a_1, a_2) \in \{(1,0), (0,1)\} \end{array} \right\}.$$

Now we will show that

$$\phi^r(g) \subseteq \left\{ \begin{array}{c} a_1(\pi_1(U,L), \pi_2(U,L)) + a_2(\pi_1(D,R), \pi_2(D,R)) + a_3(\pi_1(D,R), \pi_2(U,L)) \\ \mid \sum_{k=1}^{3} a_k = 1, a_1, a_2, a_3 \geq 0 \text{ with } a_3 = 0 \Rightarrow (a_1, a_2) \in \{(1,0), (0,1)\} \end{array} \right\}.$$

First, we show that any interior point x on $\text{co}\{(\pi_1(U,L), \pi_2(U,L)), (\pi_1(D,R), \pi_2(D,R))\}$ cannot be included in the revision equilibrium payoff set. Suppose the contrary, that is, that x is an element of the revision equilibrium payoff set. Since x can only be represented as a convex combination of $(\pi_1(U,L), \pi_2(U,L))$ and $(\pi_2(D,R), \pi_2(D,R))$, an action profile at $-T$ must put probability 1 on a profile $(\pi_1(U,L), \pi_2(U,L))$ or $(\pi_2(D,R), \pi_2(D,R))$. Since there is no public randomization device, either one of the following holds: (i) with probability one, profile (U,L) is played; (ii) with probability one, profile (D,R) is played. Without loss of generality, consider case (i).

Since x represented as a convex combination of $(\pi_1(U,L), \pi_2(U,L))$ and $(\pi_2(D,R), \pi_2(D,R))$

puts a strictly positive weight on $(\pi_2(D,R), \pi_2(D,R))$ by definition, with a positive probability there must exist a time at which profile (D,R) is played. However, this implies that, since simultaneous revision can take place with probability zero, it must be the case that there exists a time at which profile (U,R) or profile (D,L) is played. This implies that (U,R) or (D,L) must be played with a strictly positive probability at the deadline. But this contradicts our earlier conclusion that x is a convex combination only of $(\pi_1(U,L), \pi_2(U,L))$ and $(\pi_2(D,R), \pi_2(D,R))$. This completes the proof.

Finally, we show that player 1's payoff is bounded by $\pi_1(D,R)$. Suppose there exists (π_1, π_2) included in the limit set such that $\pi_1 < \pi_1(D,R)$. Consider following player 1's strategy:

- For $-t \in (-T, -t^*]$, player 1 takes U whenever she has a chance to move,
- For $-t \in (-t^*, 0]$, player 1 takes $BR_1(x)$ whenever she have a chance to move.

For sufficiently large T, player 1 can move by $-t^*$ with an arbitrarily high probability. This implies, regardless of player 2's strategy until $-t^*$, either (U,L) or (U,R) are prepared at $-t^*$ with an arbitrarily high probability. It is straightforward to show that after $-t^*$, for any subgame perfect strategy, each player takes $BR_i(x)$. This implies player 1's payoff at $-t^*$ is at least $\pi_1(D,R)$ if the state at $-t^*$ is (U,L) or (U,R). Hence for sufficiently large T, this strategy yields a payoff strictly larger than π_1. But this implies that any best response should yield a payoff equal to or more than $\pi_1(D,R)$, which implies that in any subgame perfect equilibrium player 1 obtains a payoff equal to or more than $\pi_1(D,R)$. The symmetric argument establishes that player 2's equilibrium payoff cannot be lower than $\pi_2(U,L)$.

8.2.2 Case II: $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) > 0$

The remaining case is $f^*\left(\pi, \frac{\lambda_1}{\lambda_2}\right) > 0$. The other case is completely symmetric. Further, we assume $\lambda_1 \neq \lambda_2$. When $\lambda_2 = \lambda_1$, our model allows the continuity of the cases between $\lambda_2 = \lambda_1$ and limit where λ_2 converges to λ_1 as long as $f^*(\pi, 1) \neq 0$. Therefore, the same result holds.

We use backward induction to derive the (essentially) unique subgame perfect equilibrium.[20] Let x_i^t be the prepared action by player i at $-t$. In addition, let $V_x^i(t)$ be player i's expected payoff when players prepare an action profile x at $-t$ and no player has a chance to move at $-t$. As we will prove by backward induction, since players take the Markov perfect equilibrium, $V_x^i(t)$ is a valid expression.[21]

Firstly, note that there exists $\Delta > 0$ such that, for each $-t \in (-\Delta, 0]$, given player $-i$'s prepared action x_{-i}^t, player i prepares the best response to x_{-i}^t, that is,

$$\sigma_t(x_{-i}^t) = BR(x_{-i}^t).$$

Suppose the players "know" the game proceeds as explained above after $-t$. Then, at $-t$, at each prepared action profile, each player's value is given as follows:

- At (U, L), for each i,
$$V_{UL}^i(t) = (\pi_1(U, L), \pi_2(U, L)).$$
- At (U, R), if player 2 moves first, then the payoff is $(\pi_1(U, L), \pi_2(U, L))$, if player 1 moves first, then the payoff is $(\pi_1(D, R), \pi_2(D, R))$, and otherwise, the payoff is $(\pi_1(U, R), \pi_2(U, R))$. Therefore,
$$\begin{aligned} V_{UR}^1(t) &= \underbrace{\frac{\lambda_2}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))}_{\text{player 2 moves first}}\pi_1(U, L) \\ &\quad +\underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))}_{\text{player 1 moves first}}\pi_1(D, R) \\ &\quad +\underbrace{\exp(-(\lambda_1+\lambda_2)t)}_{\text{nobody moves}}\pi_1(U, R) \end{aligned}$$

[20] All equilibria give rise to the same on-path play and even off the equilibrium path, any equilibria agree with repect to events of probability 1.

[21] Since there is only finitely many time where players are indifferent for multiple actions, we can break ties arbitrarily without loss of generality.

and

$$\begin{aligned} V_{UR}^{2}(t) &= \pi_2(U,L)\frac{\lambda_2}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t)) \\ &+\pi_2(D,R)\frac{\lambda_1}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t)) \\ &+\pi_2(U,R)\exp(-(\lambda_1+\lambda_2)t). \end{aligned}$$

- At (D,L), if player 1 moves first, then the payoff is $(\pi_1(U,L),\pi_2(U,L))$, if player 2 moves first, then the payoff is $(\pi_1(D,R),\pi_2(D,R))$, and otherwise, the payoff is $(\pi_1(D,L),\pi_2(D,L))$. Therefore,

$$\begin{aligned} V_{DL}^{1}(t) &= \frac{\lambda_1}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))\pi_1(U,L) \\ &+\frac{\lambda_2}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))\pi_1(D,R) \\ &+\exp(-(\lambda_1+\lambda_2)t)\pi_1(D,L) \end{aligned}$$

and

$$\begin{aligned} V_{DL}^{2}(t) &= \frac{\lambda_1}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))\pi_2(U,L) \\ &+\frac{\lambda_2}{\lambda_1+\lambda_2}(1-\exp(-(\lambda_1+\lambda_2)t))\pi_2(D,R) \\ &+\exp(-(\lambda_1+\lambda_2)t\pi_2(D,L)). \end{aligned}$$

- At (D,R), for each i,

$$V_{DR}^{i}(t)=(\pi_1(D,R),\pi_2(D,R))$$

Let t_1^* be the solution for

$$\pi_1(D,R)=V_{UR}^{1}(t)$$

and t_2^* be that for

$$\pi_2(U,L)=V_{UR}^{2}(t).$$

That is,

$$t_1^* = \frac{1}{\lambda_1 + \lambda_2} \log\left(\frac{\lambda_1}{\lambda_2} \frac{\pi_1(D,R) - \pi_1(U,R)}{\pi_1(U,L) - \pi_1(D,R)} + \frac{\pi_1(U,L) - \pi_1(U,R)}{\pi_1(U,L) - \pi_1(D,R)}\right)$$

and

$$t_2^* = \frac{1}{\lambda_1 + \lambda_2} \log\left(\frac{\lambda_2}{\lambda_1} \frac{\pi_2(U,L) - \pi_2(U,R)}{\pi_2(D,R) - \pi_2(U,L)} + \frac{\pi_2(D,R) - \pi_1(U,R)}{\pi_2(D,R) - \pi_2(U,L)}\right).$$

By assumption, $-t_2^* < -t_1^*$.

Then, for $-t \in (-t_1^*, 0]$, noting that $\pi_2(U,L) > V_{UR}^2(t_1^*)$ since $t_2^* > t_1^*$, we have the following:

- If $x_2^t = L$, player 1 will take U since

$$\pi_1(U,L) > V_{DL}^1(t).$$

- If $x_2^t = R$, player 1 will take D since

$$\pi_1(D,R) > V_{UR}^1(t)$$

- If $x_1^t = U$, player 2 will take L since

$$\pi_2(U,L) > V_{UR}^2(t)$$

- If $x_1^t = D$, player 2 will take R since

$$\pi_2(D,R) > V_{DL}^2(t).$$

At $-t = -t_1^*$, player 1 is indifferent between U and D when player 2 takes R.

For $-t$ slightly before $-t_1^*$, since every inequality except for $\pi_1(D,R) = V_{UR}^1(t_1^*)$ is strict at $-t_1^*$,

- If $x_2^t = L$, player 1 will take U.
- If $x_2^t = R$, player 1 will take U.
- If $x_1^t = U$, player 2 will take L.
- If $x_1^t = D$, player 2 will take R.

While players follow the above strategy, for $-t < -t_1^*$,

- If $x_2^t = L$, player 1 will take U.
- If $x_2^t = R$, player 1 will take U.
- If $x_1^t = U$, player 2 will take L. Suppose player 2 is indifferent between L and R at t. If player 2 takes L, his expected payoff is definitely $\pi_2(U, L)$. If player 2 takes R, it is a convex combination of $\pi_2(U, R)$ and $\pi_2(U, L)$, with a strictly positive weight on the former. Since $\pi_2(U, R) < \pi_2(U, L)$ by assumption, there does not exist such t.
- If $x_1^t = D$, there exists T^* such that player 2 will become indifferent between L and R.

We need to verify the fourth argument. Assuming that player 2 will take R after t, the value of taking R at $-t$ is

$$\begin{aligned} V_{DR}^2(t) &= \int_0^{t-t_1^*} \underbrace{\lambda_1 \exp(-\lambda_1 s)}_{\text{player 1 firstly moves by } t_1^*} \left\{ \begin{array}{c} \underbrace{(1-\exp(-\lambda_2(t-t_1^*-s)))}_{\text{player 2 secondly moves by } t_1^*} \pi_2(U, L) \\ +\underbrace{\exp(-\lambda_2(t-t_1^*-s))}_{\text{player 2 does not move by } t_1^*} V_{UR}^2(t_1^*) \end{array} \right\} ds \\ &\quad + \underbrace{\exp(-\lambda_1(t-t_1^*))}_{\text{player 1 does not move by } t_1^*} \pi_2(D, R) \end{aligned}$$

and the payoff of taking to L at $-t$ is

$$\underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)\right)}_{\text{player 1 firstly moves by } t_1^*}\pi_2\left(U,L\right)$$
$$+\underbrace{\int_0^{t-t_1^*}\underbrace{\exp(-\lambda_1 s)}_{\text{player 1 does not move by } s}\underbrace{\lambda_2\exp(-\lambda_2 s)}_{\text{player 2 moves at } s}V_{DR}^2(t-s)ds}_{\text{player 2 firstly moves by } t_1^*}$$
$$+\underbrace{\exp(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right))}_{\text{nobody moves by } t_1^*}V_{DL}^2\left(t_1^*\right).$$

Note that

$$\begin{aligned}
V_{DR}^2\left(t\right) &= \int_0^{t-t_1^*}\lambda_1\exp(-\lambda_1 s)\pi_2\left(U,L\right)ds \\
&\quad -\int_0^{t-t_1^*}\lambda_1\exp\left(-\lambda_1 s-\lambda_2\left(t-t_1^*-s\right)\right)\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right)ds \\
&\quad +\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\pi_2\left(D,R\right) \\
&= \left(1-\exp(-\lambda_1\left(t-t_1^*\right))\right)\pi_2\left(U,L\right) \\
&\quad -\lambda_1\frac{\exp\left(-\lambda_1\left(t-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-t_1^*\right)\right)}{\lambda_2-\lambda_1}\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right) \\
&\quad +\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\pi_2\left(D,R\right).
\end{aligned}$$

and the payoff of taking L is

$$\begin{aligned}
& \frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)\right)\pi_2\left(U,L\right) \\
& +\int_0^{t-t_1^*}\lambda_2\exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\left(\begin{array}{c}\left(1-\exp(-\lambda_1\left(t-s-t_1^*\right))\right)\pi_2\left(U,L\right) \\ -\lambda_1\frac{\exp\left(-\lambda_1\left(t-s-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-s-t_1^*\right)\right)}{\lambda_2-\lambda_1}\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right) \\ +\exp\left(-\lambda_1\left(t-s-t_1^*\right)\right)\pi_2\left(D,R\right)\end{array}\right)ds \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)V_{DL}^2\left(t_1^*\right) \\
= & \frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)\right)\pi_2\left(U,L\right) \\
& +\left(\frac{\lambda_2}{\lambda_1+\lambda_2}-\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\right)\pi_2\left(U,L\right) \\
& -\exp(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right))\left(\frac{\lambda_2}{\lambda_1+\lambda_2}-1\right)\pi_2\left(U,L\right) \\
& +\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\left(\begin{array}{c}\frac{\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)-\exp\left(-\lambda_1\left(t-t_1^*\right)\right)}{\lambda_2} \\ -\frac{\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-t_1^*\right)\right)}{\lambda_1}\end{array}\right)\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right) \\
& -\left(\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)-\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\right)\pi_2\left(D,R\right) \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)V_{DL}^2\left(t_1^*\right) \\
= & \left(1-\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\right)\pi_2\left(U,L\right) \\
& -\left(\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)+\frac{\lambda_1\exp\left(-\lambda_1\left(t-t_1^*\right)\right)-\lambda_2\exp\left(-\lambda_2\left(t-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right)\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right) \\
& -\left(\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)-\exp\left(-\lambda_1\left(t-t_1^*\right)\right)\right)\pi_2\left(D,R\right) \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-t_1^*\right)\right)V_{DL}^2\left(t_1^*\right)
\end{aligned}$$

- The difference is given by

$$\left\{\begin{array}{c}\left(\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right)+\left(\pi_2\left(D,R\right)-V_{DL}^2\left(t_1^*\right)\right)\right)\exp\left(-\lambda_1\left(t-t_1^*\right)\right) \\ -\left(\pi_2\left(U,L\right)-V_{UR}^2\left(t_1^*\right)\right)\end{array}\right\}\exp\left(-\lambda_2\left(t-t_1^*\right)\right)$$

  Therefore, since $\pi_2\left(D,R\right)-V_{DL}^2\left(t_1^*\right)>0$, there exists unique T^* such that the difference is positive for $-t\in(-T^*,t_1^*]$, zero for $-t=-T^*$, and negative for $-t<-T^*$.

Therefore, for $-t\in(-T^*,-t_1^*)$, we have

- If $x_2^t = L$, player 1 will take U.
- If $x_2^t = R$, player 1 will take U.
- If $x_1^t = U$, player 2 will take L.
- If $x_1^t = D$, player 2 will take R.

Since every incentive except for player 2 with $x_1 = D$ is strict for $-t$ slightly before $-T^*$, the equilibrium strategy at $-t$ is

- If $x_2^t = L$, player 1 will take U.
- If $x_2^t = R$, player 1 will take U.
- If $x_1^t = U$, player 2 will take L.
- If $x_1^t = D$, player 2 will take L.

As long as players follow the above strategy,

- If $x_2^t = L$, player 1 will take U.
- If $x_2^t = R$, player 1 will take U
- If $x_1^t = U$, player 2 will take L since, assuming player 2 will take L after that,

$$\begin{aligned} & V_{UL}^2(t) \\ = \; & \pi_2(U, L) \\ > \; & \underbrace{(1 - \exp(-\lambda_2(t - t_1^*)))}_{\text{player 2 can move by } t_1^*} \pi_2(U, L) + \exp(-\lambda(t - t_1^*)) V_{UR}^2(t_1^*) \\ = \; & V_{UR}^2(t). \end{aligned}$$

since

$$\pi_2(U, L) > V_{UR}^2(t_1^*).$$

- If $x_1 = D$, player 2 will take L from above.

  We need to verify the second argument: at $-t$, if players follow the strategy above afterwards, the payoff of taking U is given by

$$V_{UR}^{1}(t) = \underbrace{\left(1 - \exp\left(-\lambda_2\left(t - t_1^*\right)\right)\right)}_{\text{player 2 can move by } t_1^*} \pi_1(U, L) + \exp\left(-\lambda_2\left(t - t_1^*\right)\right) V_{UR}^{1}\left(t_1^*\right).$$

On the other hand, the payoff of taking D is given by

$$
\begin{aligned}
&\int_0^{t-T^*} \underbrace{\exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\lambda_1}_{\substack{\text{player 1 moves first by } T^*, \\ \text{which induces } (U,R)}} V_{UR}^1\left(t-s\right)ds \\
&+\int_0^{t-T^*} \underbrace{\exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\lambda_2}_{\substack{\text{player 2 moves first by } T^*, \\ \text{which induces } (D,L)}} \times \\
&\left[\begin{array}{c}
\underbrace{\left(1-\exp\left(-\lambda_1\left(t-T^*-s\right)\right)\right)}_{\substack{\text{player 1 moves to by } T^*, \\ \text{which induces } (U,L)}}\pi_1\left(U,L\right) \\
+\underbrace{\exp\left(-\lambda_1\left(t-T^*-s\right)\right)}_{\substack{\text{player 1 does not move by } T^*, \\ \text{which induces } (D,L)}}\times \\
\left\{\begin{array}{c}
\underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right)}_{\substack{\text{player 1 moves first by } t_1^*, \\ \text{which induces } (U,L)}}\pi_1\left(U,L\right) \\
+\int_0^{T^*-t_1^*}\left(\begin{array}{c}
\underbrace{\exp\left(-\left(\lambda_1+\lambda_2\right)\tau\right)\lambda_2}_{\substack{\text{player 2 moves first by } t_1^*, \\ \text{which induces } (D,R)}} \\
\times\left(\begin{array}{c}
\underbrace{\exp\left(-\lambda_1\left(T^*-t_1^*-\tau\right)\right)}_{\substack{\text{player 1 does not move until } t_1^*, \\ \text{which induces } (D,R)}}\pi_1\left(D,R\right) \\
+\int_0^{T^*-t_1^*-\tau}\underbrace{\exp\left(-\lambda_1\theta\right)\lambda_1}_{\substack{\text{player 1 moves by } t_1^*, \\ \text{which induces } (D,R)}}V_{UR}^1\left(T^*-\tau-\theta\right)d\theta
\end{array}\right)
\end{array}\right)d\tau \\
+\underbrace{\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)}_{\substack{\text{nobody moves by } t_1^*, \\ \text{which induces } (D,L)}}V_{DL}^1\left(t_1^*\right)
\end{array}\right\}
\end{array}\right]ds \\
&+\underbrace{\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)}_{\substack{\text{nobody moves by } T^*, \\ \text{which induces } (D,R)}}\left\{\begin{array}{c}
\underbrace{\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)}_{\substack{\text{player 1 does not move by } t_1^*, \\ \text{which induces } (D,R)}}\pi_1\left(D,R\right) \\
+\int_0^{T^*-t_1^*}\underbrace{\exp\left(-\lambda_1\tau\right)\lambda_1}_{\substack{\text{player 1 moves by } t_1^*, \\ \text{which induces } (U,R)}}V_{UR}^1\left(T^*-\tau\right)d\tau
\end{array}\right\}.
\end{aligned}
$$

The first term is

$$
\begin{aligned}
& \pi_1(U,L)\int_0^{t-T^*}\lambda_1\exp\left(-\left(\lambda_1+\lambda_2\right)s\right) \\
& -\left(\pi_1(U,L)-V_{UR}^1\left(t_1^*\right)\right)\int_0^{t-T^*}\lambda_1\exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\exp\left(-\lambda_2\left(t-t_1^*-s\right)\right)ds \\
= & \frac{\lambda_1}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right)\pi_1(U,L) \\
& -\left(\pi_1(U,L)-V_{UR}^1\left(t_1^*\right)\right)\left(\exp\left(-\lambda_2\left(t-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right)\right)
\end{aligned}
$$

and the second term is

$$
\begin{aligned}
& \pi_1(U,L)\frac{\lambda_2}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& +\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& \times\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\pi_1(U,L) \\ +\left(\pi_1(U,L)-\pi_1(D,R)\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\end{array}\right\}.
\end{aligned}
$$

The calculation goes as follows: the integral with respect to θ yields

$$
\begin{aligned}
& \int_0^{T^*-t_1^*-\tau}\exp\left(-\lambda_1\theta\right)\lambda_1 V_{UR}^1\left(T^*-\tau-\theta\right)d\theta \\
= & \int_0^{T^*-t_1^*-\tau}\exp\left(-\lambda_1\theta\right)\lambda_1\pi_1(U,L) \\
& -\left(\pi_1(U,L)-V_{UR}^1\left(t_1^*\right)\right)\int_0^{T^*-t_1^*-\tau}\lambda_1\exp\left(-\lambda_1\theta\right)\exp\left(-\lambda_2\left(T^*-\tau-\theta-t_1^*\right)\right)d\theta \\
= & \pi_1(U,L)\left(1-\exp\left(\lambda_1\left(T^*-t_1^*-\tau\right)\right)\right) \\
& -\left(\pi_1(U,L)-V_{UR}^1\left(t_1^*\right)\right)\frac{\lambda_1}{\lambda_2-\lambda_1}\left(\exp\left(-\lambda_1\left(T^*-t_1^*-\tau\right)\right)-\exp\left(-\lambda_2\left(T^*-t_1^*-\tau\right)\right)\right).
\end{aligned}
$$

Substituting this to the integral with respect to τ yields

$$\int_0^{T^*-t_1^*} \exp\left(-\left(\lambda_1+\lambda_2\right)\tau\right)\lambda_2 \left(\begin{array}{c} \pi_1\left(U,L\right) \\ -\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\exp\left(-\lambda_1\left(T^*-t_1^*-\tau\right)\right) \\ -\left(\pi_1\left(U,L\right)-V_{UR}^1\left(t_1^*\right)\right)\frac{\lambda_1}{\lambda_2-\lambda_1}\left(\begin{array}{c} \exp\left(-\lambda_1\left(T^*-t_1^*-\tau\right)\right) \\ -\exp\left(-\lambda_2\left(T^*-t_1^*-\tau\right)\right) \end{array}\right) \end{array}\right) d\tau$$

$$\begin{aligned}
= \;& \pi_1\left(U,L\right)\frac{\lambda_2}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right) \\
& +\pi_1\left(U,L\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \\
& -\pi_1\left(D,R\right)\left(\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)\right) \\
& -V_{UR}^1\left(t_1^*\right)\left(\begin{array}{c} \frac{-\lambda_1\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\lambda_2\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \\ -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right) \end{array}\right) \\
= \;& \pi_1\left(U,L\right)\frac{\lambda_2}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right) \\
& +\pi_1\left(U,L\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \\
& -\pi_1\left(D,R\right)\left(\begin{array}{c} \exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right) \\ -\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right) \\ +\frac{-\lambda_1\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\lambda_2\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \\ -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right) \end{array}\right) \\
& \left(\text{since } V_{UR}^1\left(t_1^*\right)=\pi_1\left(D,R\right)\right) \\
= \;& \pi_1\left(U,L\right)\frac{\lambda_2}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right) \\
& +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}.
\end{aligned}$$

Substituting this into the integral with respect to s yields

$$
\begin{aligned}
& \int_0^{t-T^*} \exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\lambda_2 \\
& \times \left[\begin{array}{c} \left(1-\exp\left(-\lambda_1\left(t-T^*-s\right)\right)\right)\pi_1\left(U,L\right) \\ +\exp\left(-\lambda_1\left(t-T^*-s\right)\right)\left\{\begin{array}{c} \frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right)\pi_1\left(U,L\right) \\ +\frac{\lambda_2}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\right)\pi_1\left(U,L\right) \\ +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \\ +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)V_{DL}^1\left(t_1^*\right) \end{array}\right\} \end{array}\right] ds \\
= \; & \pi_1\left(U,L\right)\int_0^{t-T^*} \exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\lambda_2 ds \\
& +\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right) \\ +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \end{array}\right\}\times \\
& \int_0^{t-T^*} \exp\left(-\left(\lambda_1+\lambda_2\right)s\right)\lambda_2\exp\left(-\lambda_1\left(t-T^*-s\right)\right) \\
= \; & \pi_1\left(U,L\right)\frac{\lambda_2}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& +\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& \times\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-V_{DL}^1\left(t_1^*\right)\right) \\ +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)+\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} \end{array}\right\}
\end{aligned}
$$

The third term is

$$
\begin{aligned}
& \exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left\{\begin{array}{c}\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)\pi_1\left(D,R\right) \\ +\int_0^{T^*-t_1^*}\exp\left(-\lambda_1\tau\right)\lambda_1 V_{UR}^1\left(T^*-\tau\right)d\tau\end{array}\right\} \\
= & \exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)\pi_1\left(D,R\right) \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\begin{array}{c}\pi_1\left(U,L\right)\left(1-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)\right) \\ +\frac{\lambda_1}{\lambda_2-\lambda_1}\left(\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)\right) \\ \times\left(V_{UR}^1\left(t_1^*\right)-\pi_1\left(U,L\right)\right)\end{array}\right) \\
= & \exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\pi_1\left(U,L\right) \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right) \\
& \times\left(\pi_1\left(D,R\right)-\pi_1\left(U,L\right)\right) \\
& \text{since }\left(V_{UR}^1\left(t_1^*\right)=\pi_1\left(D,R\right)\right)
\end{aligned}
$$

since

$$
\begin{aligned}
& \int_0^{T^*-t_1^*}\exp\left(-\lambda_1\tau\right)\lambda_1 V_{UR}^1\left(T^*-\tau\right)d\tau \\
= & \pi_1\left(U,L\right)\int_0^{T^*-t_1^*}\exp\left(-\lambda_1\tau\right)\lambda_1 d\tau \\
& +\left(V_{UR}^1\left(t_1^*\right)-\pi_1\left(U,L\right)\right)\int_0^{T^*-t_1^*}\exp\left(-\lambda_1\tau\right)\lambda_1\exp\left(-\lambda_2\left(T^*-\tau-t_1^*\right)\right)d\tau \\
= & \pi_1\left(U,L\right)\left(1-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)\right) \\
& +\left(V_{UR}^1\left(t_1^*\right)-\pi_1\left(U,L\right)\right)\frac{\lambda_1}{\lambda_2-\lambda_1}\left(\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)\right).
\end{aligned}
$$

Therefore, in total, the value of taking D is

$$
\begin{aligned}
& \frac{\lambda_1}{\lambda_2+\lambda_1}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right)\pi_1\left(U,L\right) \\
& -\left(\pi_1\left(U,L\right)-V_{UR}^1\left(t_1^*\right)\right)\left(\exp\left(-\lambda_2\left(t-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right)\right) \\
& +\pi_1\left(U,L\right)\frac{\lambda_2}{\lambda_1+\lambda_2}\left(1-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& +\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& \times\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-V_{DL}^1\left(t_1^*\right)\right) \\ +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\end{array}\right\} \\
& +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\pi_1\left(U,L\right) \\
& -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right) \\
& \times\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right) \\
= \; & \pi_1\left(U,L\right) \\
& -\left(\pi_1\left(U,L\right)-V_{UR}^1\left(t_1^*\right)\right)\left(\exp\left(-\lambda_2\left(t-t_1^*\right)\right)-\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right)\right) \\
& +\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& \times\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-V_{DL}^1\left(t_1^*\right)\right) \\ +\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\lambda_2\frac{\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\end{array}\right\} \\
& -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right) \\
& \times\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)
\end{aligned}
$$

Subtracting $V_{UR}^1\left(t\right)$ yields

$$
\begin{aligned}
& \left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\left(\begin{array}{c}\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right) \\ -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right)\end{array}\right) \\
& +\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) \\
& \times\left\{\begin{array}{c} -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-V_{DL}^1\left(t_1^*\right)\right) \\ +\lambda_2\frac{\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\left(\pi_1\left(U,L\right)-\pi_1\left(D,R\right)\right)\end{array}\right\}.
\end{aligned}
$$

Since

$$\begin{aligned}
\left(\exp\left(-\lambda_1\left(t-T^*\right)\right)-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\right) &> 0 \\
-\exp\left(-\left(\lambda_1+\lambda_2\right)\left(T^*-t_1^*\right)\right)\left(\pi_1\left(U,L\right)-V_{DL}^1\left(t_1^*\right)\right) &< 0 \\
\frac{\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)-\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1} &< 0,
\end{aligned}$$

it suffices to show that $F\left(t\right)\leq 0$ with

$$\begin{aligned}
F\left(t\right) \ : &= \exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right) \\
&\quad -\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right)
\end{aligned}$$

At the limit where t goes to infinity, we have

$$\lim_{t\to\infty}F\left(t\right)=0.$$

Hence, it suffices to show that

$$F'\left(t\right)>0.$$

Since

$$\begin{aligned}
\frac{F'\left(t\right)}{\lambda_1+\lambda_2} &= -\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right) \\
&\quad +\exp\left(-\left(\lambda_1+\lambda_2\right)\left(t-T^*\right)\right)\left(\frac{\lambda_2\exp\left(-\lambda_1\left(T^*-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_2\left(T^*-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right) \\
&= -\exp\left(-\lambda_2\left(t-t_1^*\right)-\lambda_1\left(t-T^*\right)\right) \\
&\quad +\left(\frac{\lambda_2\exp\left(-\lambda_2\left(t-T^*\right)-\lambda_1\left(t-t_1^*\right)\right)-\lambda_1\exp\left(-\lambda_1\left(t-T^*\right)-\lambda_2\left(t-t_1^*\right)\right)}{\lambda_2-\lambda_1}\right),
\end{aligned}$$

if $\lambda_2 > \lambda_1$, we have

$$\begin{aligned}
\frac{(\lambda_2 - \lambda_1)}{\lambda_1 + \lambda_2} F'(t) &= -\lambda_2 \exp\left(-\lambda_2 (t - t_1^*) - \lambda_1 (t - T^*)\right) + \lambda_1 \exp\left(-\lambda_2 (t - t_1^*) - \lambda_1 (t - T^*)\right) \\
&\quad + \lambda_2 \exp\left(-\lambda_2 (t - T^*) - \lambda_1 (t - t_1^*)\right) - \lambda_1 \exp\left(-\lambda_1 (t - T^*) - \lambda_2 (t - t_1^*)\right) \\
&= -\lambda_2 \exp\left(-\lambda_2 (t - t_1^*) - \lambda_1 (t - T^*)\right) + \lambda_2 \exp\left(-\lambda_1 (t - t_1^*) - \lambda_2 (t - T^*)\right) > 0
\end{aligned}$$

as desired. Symmetrically, if $\lambda_2 < \lambda_1$, we have

$$\frac{(\lambda_2 - \lambda_1)}{\lambda_1 + \lambda_2} F'(t) < 0.$$

References

[1] **Abreu, Dilip and Faruk Gul** "Bargaining and Reputation" *Econometrica*, 2000, *68*, 85-117.

[2] — **and David Pearce** "Bargaining, Reputation and Equilibrium Selection in Repeated Games" *Princeton Economic Theory Papers*, 2000.

[3] **Ambrus, Attila and Shih-En Lu**, "A continuous-time model of multilateral bargaining," 2009, *mimeo.*

[4] **Benoit, Jean-Pierre and Vijay Krishna** "Finitely Repeated Games," *Econometrica*, 1985, *53*, 905-922.

[5] **Calcagno, Riccardo and Stefano Lovo** "Preopening and Equilibrium Selection," *mimeo.*

[6] **Carlsson, Hans and Eric van Damme** "Global Games and Equilibrium Selection," *Econometrica*, 1993, *61*, 989-1018.

[7] **Caruana, Guillermo and Liran Einav** "A Theory of Endogenous Commitment," *Review of Economic Studies*, 2008, *75*, 99-116.

[8] **Dutta, Prajit K.** "A Folk Theorem for Stochastic Games," *Journal of Economic Theory*, 1995, 66, 1-32.

[9] **Dutta, Prajit K.** "Coordination Need Not Be A Problem," 2003, mimeo.

[10] **Farrell, J., and Saloner, G.**, "Standardization, Compatibility, and Innovation," *RAND Journal of Economics*, 1985, *16*, 70-83.

[11] **Fudenberg, Drew and Wojciech Olszewski** "Repeated Games with Asynchronous Monitoring" 2009, mimeo.

[12] **Harsanyi, John, C. and Reinhard Selten** "A general Theory of Equilibrium Selection in Games," MIT press, Cambridge, MA. 1988.

[13] **Kamada, Yuichiro and Michihiro Kandori**, "Revision Games," 2009, mimeo.

[14] **— and Takuo Sugaya**, "Policy Announcement Game: Valence Candidates and Ambiguous Policies," 2010a, mimeo.

[15] **Kandori, Michihiro, George J. Mailath, and Rafael Rob** "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, 1993, *61*, 29-56.

[16] **Lipman, Barton, L. and Ruqu Wang** "Switching Costs in Frequently Repeated Games," *Journal of Economic Theory*, 2000, *93*, 149-190.

[17] **Matsui, Akihiko and Kiminori Matsuyama** "An Approach to Equilibrium Selection," *Journal of Economic Theory*, 1995, *65*, 415-434.

[18] **Morris, Stephen, Rafael Rob, and Hyun Song Shin** "p-Dominance and Belief Potential," *Econometrica*, 1995, *63*, 145-167.

[19] **— and Hyun Song Shin** "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks" *American Economic Review*, 1998, *88*, 587-597.

[20] **Oyama, Daisuke, Satoru Takahashi, and Josef Hofbauer** "Monotone Methods for Equilibrium Selection under Perfect Foresight Dynamics," *Theoretical Economics*, 2008, *3*, 155-192.

[21] **Rubinstein, Ariel** "The Electronic Mail Game: A Game with Almost Common Knowledge," *American Economic Review*, 1989, *79*, 389-391.

[22] **Sugaya, Takuo and Satoru Takahashi** "Coordination Failure in Repeated Games with Private Monitoring," 2009, mimeo

[23] **Takahashi**, **Satoru** "Infinite Horizon Common Interest Games with Perfect Information," *Games and Economic Behavior*, 2005, *53*, 231-247.

[24] **Young, H. Peyton** "The Evolution of Conventions," *Econometrica*, 1993, *61*, 57-84.

[25] — "Conventional Contracts," *Review of Economic Studies*, 1998, *65*, 773-792.