ONLINE APPENDIX to: Revision Games

Part II: Applications and Robustness

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B Detailed Analysis of the Bertrand Competition Component Game

Nash and fully collusive prices

To understand the setting, first note that the symmetric Nash price is

$$p^N = \begin{cases} c & \text{if} \quad c < \frac{2}{3}v \\ v - \frac{c}{2} & \text{if} \quad c \in [\frac{2}{3}v, v] \\ \frac{v}{2} & \text{if} \quad v < c \end{cases} .$$

There are three cases for the following reasons: First, when c is very high ($v < c$), the firms do not serve the buyers in $\left(\frac{v}{2c}, 1 - \frac{v}{2c}\right)$, and each firm acts as if they were a monopoly. If c is not too high ($c \leq v$), a marginal price cut decreases the market share of the opponent, and the property of the Nash equilibrium depends on the relative sizes of c and v.

To see this, it is easy to first consider the situation where v is very high (specifically, the following discussion is for the case $c < \frac{2}{3}v$). In such a case, the Bertrand-type competition implies that the Nash price is low, while a Nash price is higher than the marginal cost (which is zero) because of product differentiation. The discrepancy between the marginal cost and the Nash price depends on the degree of product differentiation, and the Nash price is exactly c in our linear-transportation-cost model. The reason that the Nash price is increasing in the transportation cost is that if the transportation cost is low, firms can decrease the price only a little bit to increase the market share significantly, so it is hard to sustain a high price. This feature of low transportation cost is going to be important in characterizing the optimal plan for the case with low transportation cost. We will be detailed on this in what follows.

When v is small (specifically, the following discussion is for the case $c \in [\frac{2}{3}v, v]$), however, even if firms quote price c, not every customers make purchase. If this is the case, then each firm has an incentive to further cut their prices to induce all buyers to purchase. This is why the symmetric Nash price for low c is exactly the value v minus the transportation cost of the buyer at position $\frac{1}{2}$ (which is $c \times \frac{1}{2}$).[1]

[1] Note that there are multiple asymmetric equilibria in this case (i.e., when $c \in [\frac{2}{3}v, v]$) corresponding to various possible splits of the market share.

Now, consider the fully collusive price:

$$p^* = \begin{cases} v - \frac{c}{2} & \text{if} \quad c \leq v \\ \frac{v}{2} & \text{if} \quad v < c \end{cases}.$$

When $v < c$, the fully collusive price is $\frac{v}{2}$ for the same reason as for the Nash price. For the other case, it is optimal to divide the customer base in halves, inducing all customers to buy. That is, the fully collusive price is exactly at the price that makes the middle customer (at 0.5) indifferent between purchasing and not. Such a price increases as the transportation cost decreases, and this is why, for $c \leq v$, the fully collusive price increases as the product differentiation decreases.

In total, when $\frac{2}{3}v \leq c$, the unique Nash price profile coincides with the fully collusive profile, so there is no room for nontrivial cooperation. The intuition is that the cost of transportation is so high that a firm's marginal price cutting at the fully collusive price profile does not induce enough buyers to switch to the firm. For this reason, hereafter we assume that $c \in [0, \frac{2}{3}v)$. In this case, $\pi^N = \frac{c}{2}$ and $\pi(p) = \frac{p}{2}$.

Gain from deviation

The gain from deviation $d(p)$ depends on relative values of c and p. If c is high relative to p, then the static best response is not to steal all the buyers, but to trade-off the decrease of the price and the increase of the customer base. This should be clear when the price is very close to the Nash price, which is when the best response is to set a price not too far from the Nash price, and cutting the price to steal all the customers is obviously suboptimal. On the other hand, if c is relatively low, then an excessive price cut is not necessary to steal all the customer base, so the static best response is to drop the price to the one that is just enough to serve all the customers.

Since the range over which p moves is dictated by the size of v, whether stealing all the customers can be a static best response on the equilibrium plan depends on the relative values of c and v. Specifically, if $c \in (\frac{2}{7}v, \frac{2}{3}v)$, then there is no such possibility, and the static best response always solves the first order condition, and a calculation shows that $BR(p) = \frac{p+c}{2}$ and $d(p) = \frac{(p-c)^2}{8c}$. If, however, $c \in (0, \frac{2}{7}v]$, then there is a possibility for the static best response to be stealing all customers when p is high. In other words, the degree of differentiation is so small that when the opponent sets a price close to the fully collusive price, the static best reply is to set a price just enough to attract all the buyers, that is, $BR(p) = p - c$ and hence $d(p) = \frac{p}{2} - c$.

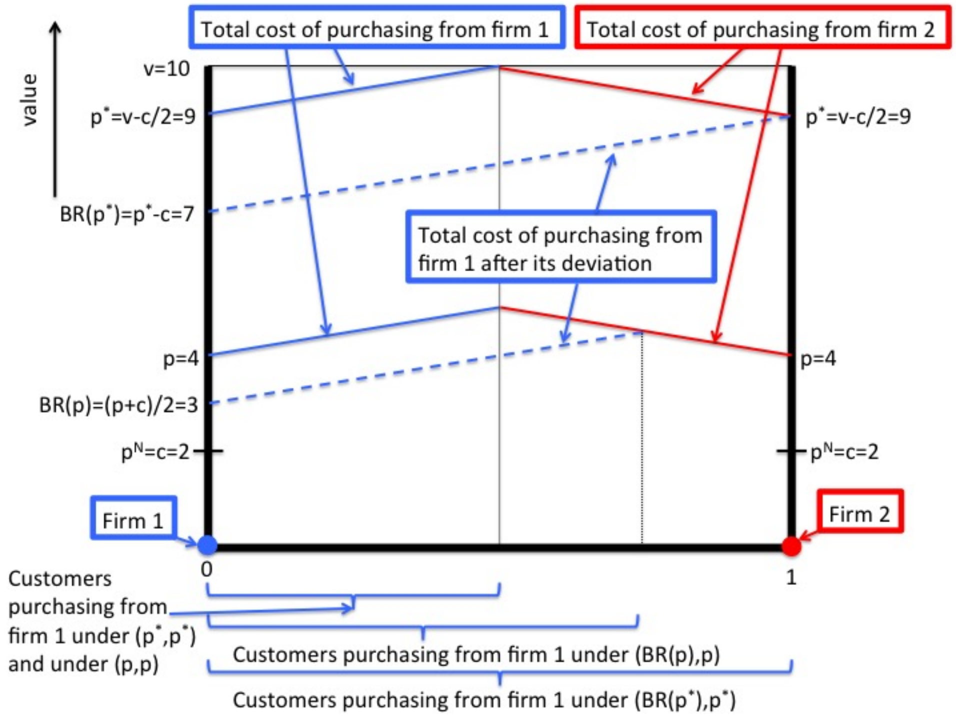

Figure 1: Low product differentiation: $v = 10$ and $c = 2$ (hence $c \in (0, \frac{2}{7}v])$).

Figures 1 and 2 illustrate this difference by considering the cases with $c \in (0, \frac{2}{7}v]$ and $c \in (\frac{2}{7}v, \frac{2}{3}v)$, respectively. In the former case depicted in Figure 1, when the price is as high as the optimal price p^*, the static best response $BR(p^*)$ is such that the resulting price profile $(BR(p^*), p^*)$ gives the deviating firm (firm 1 in the figure) the market share of 1. However, if the price is low (the case depicted as $p = 4$), then the best response has to balance the increase in the market share and the decrease of the price. In the latter case depicted in Figure 2, even at the optimal price p^*, the best response $BR(p^*)$ does not give the deviating firm the market share of 1. If the firm were to set a price such that it steals the whole market (to set the price $p = 2.5$ in the example), then the firm's profit would be only as high as the Nash profit (i.e., both firms pricing at $p = p^N$).

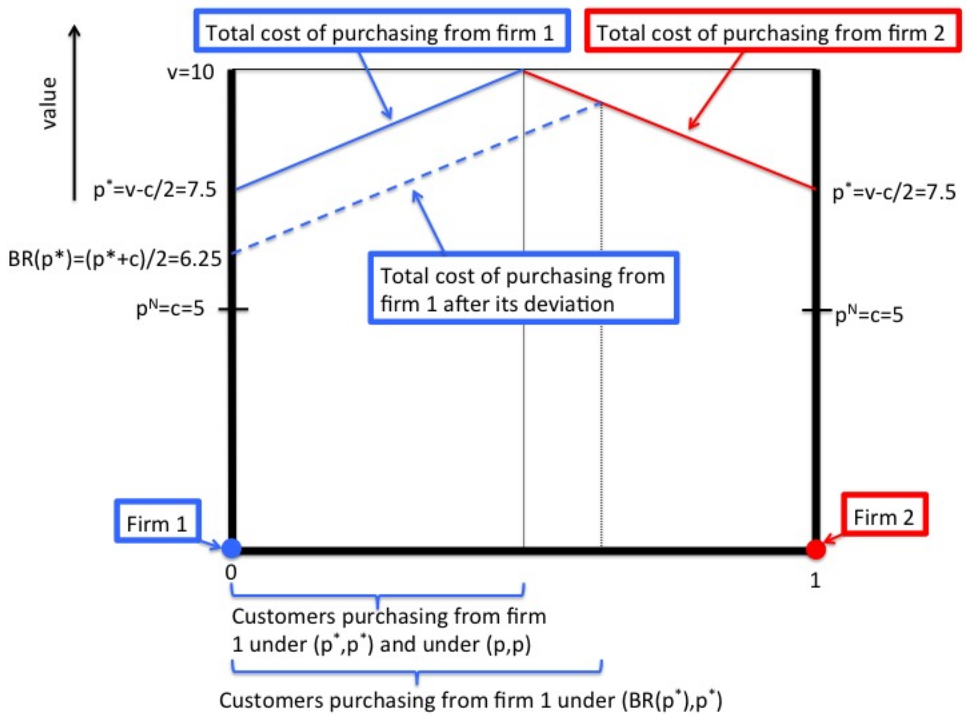

Figure 2: High product differentiation: $v = 10$ and $c = 5$ (hence $c \in (\frac{2}{7}, \frac{2}{3}v)$).