VOTER PREFERENCES, POLARIZATION, AND ELECTORAL POLICIES*

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Abstract

In most variants of the Hotelling-Downs model of election, it is assumed that voters have concave utility functions. This assumption is arguably justified in issues such as economic policies, but convex utilities are perhaps more appropriate in others such as moral or religious issues. In this paper we analyze the implications of convex utility functions in a two-candidate probabilistic voting model with a polarized voter distribution. We show that the equilibrium policies diverge if and only if voters' utility function is sufficiently convex. If two or more issues are involved, policies converge in "concave issues" and diverge in "convex issues."

1 Introduction

A standard practice in the literature of electoral competition a la Hotelling-Downs is to assume that voters have concave utility function that depends on the distance between their bliss points and the realized policy (Hotelling (1929), Downs (1957) and Black (1958)). However, in reality, voters seem to have different patterns of preferences given different political issues, and the assumption of concavity is likely realistic only in certain issues.

In this paper we argue that a variety of voter preferences naturally arise in electoral situations, and show that different patterns of voter preferences have different implications on electoral outcomes. In particular, announced policies can diverge if voter utility function is convex with respect to the distance between the bliss point and the policy, while concavity implies convergence.

Because most existing studies on spatial models of elections assume that voters' utility functions are concave, convex utility functions might seem to be unrealistic at first glance (even though the assumption of single-peaked preferences is still preserved throughout this paper).[1] Justification for concave utility functions, however, seems unclear. A skeptical view has been eloquently expressed by Osborne (1995) as follows:

> *The assumption of concavity is often adopted, first because it is associated with 'risk aversion' and second because it makes it easier to show that an equilibrium exists. However, I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (a view that seems to be shared by Downs 1957, 119-20). ... Further, it is not clear that evidence that people are risk averse in economic decision-making has any relevance here. I conclude that in the absence of any convincing empirical evidence, it is not clear which of the assumptions is more appropriate.*

Indeed, non-concave utility functions are used extensively in the empirical literature. Poole and Rosenthal (1997), for instance, argue that utility functions that are not concave may fit the data better.[2] In the theoretical literature, although concavity is often assumed, Shepsle (1972) and Aragones and Postlewaite (2002) allow for convex utility functions, relating them to the "intensity" of voter preferences. We also believe utility functions that are not concave are sometimes plausible and useful in the voting context and pursue the implications different properties of voters' preferences have for the candidates' policy positions. However, to the best of our knowledge, neither Osborne (1995) nor any subsequent work has found a link between convexity and policy divergence studied in this paper.

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[1]Convexity of the voter utility function that we employ in this paper does not imply the failure of single-peakedness, although the latter might easily lead to divergent policies in equilibrium. Throughout the paper we retain the assumption that the voter preferences are single-peaked, following the standard assumption in the literature.

[2]According to Poole and Rosenthal (1997), a bell-shaped function fits the data better than a concave function. As a bell-shaped function is composed of concave and convex parts, their study suggests studying implications of convexity.

To highlight the effect of voter preferences, this paper considers a probabilistic voting model that is kept simple and tractable. As in the standard probabilistic voting model, a voter votes for a candidate whose announced policy is closer to her bliss point with higher probability, but not with probability one. The probability of voting for the closer candidate is increasing in the utility difference between the two announced policies.[3] While our setup closely follows that of standard models, we employ one novel feature. Specifically, in our model voters' utility function is not necessarily concave.

Our main finding is an unexpected relationship between voter preferences and equilibrium policies. More specifically, when voters are more polarized than in the uniform distribution, we show that there is a convexity threshold of voters' utility function (i) below which policy convergence is a unique equilibrium, and (ii) above which policy divergence is a unique equilibrium. As the voters become more polarized, divergent policies prevail in equilibrium for a wider range of voter utility functions. Moreover, we find that social welfare is maximized in each divergent equilibrium, but not necessarily in every convergent equilibrium.

Economic policy is arguably a concave issue, given the evidence that individuals are risk averse in financial decisions. By contrast, voters may have convex utility functions on moral or religious issues such as same-sex marriage. For example, when a civil union law was introduced in Connecticut in 2005, Anne Stanback, president of a gay rights advocacy organization Love Makes a Family, commented "It's bittersweet" as it was a move in the right direction but did not go far enough (*Boston Globe*, April 21, 2005). The attitude of gay people to a State Supreme Court decision was in a sharp contrast, when it ruled to legalize same-sex marriages in 2008: "The opinion in Connecticut was hailed by jubilant gay couples and their advocates" (*New York Times*, October 11, 2008). This difference is remarkable, given that laws defining civil unions provide most of the legal benefits of marriage in all but name "marriage." Indeed, the majority opinion in the State Supreme Court declares "marriage and civil unions do embody the same legal rights under our law." Even so, the Court's majority opinion seems to have recognized the large utility difference for gay couples between a civil union and a marriage, as it continues to write "they [marriage and civil unions] are by no means equal" and rules in favor of gay couples. The strong dissatisfaction of gay couples regarding civil unions and

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[3]There are a number of "microfoundations" of why voters may vote randomly, some of which we explain in Section 2.

the contrasting happiness they feel about traditional marriage suggest a convex utility function, meaning that voters have strong feelings regarding policy changes around their bliss points. Abortion may provide another example of convex utility functions: A pro-life activist may equate abortion with murder and find it (almost) equally abhorrent, even if it is conducted at an early stage of pregnancy.

While the anecdotes above are suggestive at best and they do not invalidate the usefulness of the traditional election models in any way, we believe they still suggest a new approach to understanding electoral competition.[4] Recall that our theory predicts that politicians tend to converge on concave issues and diverge on convex issues. This prediction is consistent with the observed pattern of policies in American politics, where candidates take similar positions on some issues such as economic policy, while they differ sharply on other issues, especially on those with moral or religious content. For example, the 2008 Democratic National Platform declared that they "support the full inclusion of all families, including same-sex couples, in the life of our nation, and support equal responsibility, benefits, and protections," and "oppose the Defense of Marriage Act."[5] The Republican platform in the same year described the Democrats' opposition to this act as unbelievable, and wrote that they "call for a constitutional amendment that fully protects marriage as a union of a man and a woman."[6]

Our policy divergence result is novel and it may appear surprising, especially given that voters have single-peaked preferences and politicians are office-motivated in our model.

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[4]One question that has sometimes been raised to us is why we do not change the way we measure the distance in the policy space instead of changing the utility function of voters, say by assuming that the "middle" policy is closer to the rightist policy than the leftist policy. Our answer is that such an exercise needs a substantial departure from the standard Hotelling-Downs framework and is potentially intractable. For example, suppose that voters have a linear utility function and we move the "middle" policy to the right. Then for left-wing voters, the utility difference between the middle policy and their least preferred one becomes smaller than the difference between their most preferred policy and the middle one (thus this movement of the middle policy would correspond to assuming convex utility in our model). This implies, however, that for the right-wing voters the utility difference between the middle policy and their least preferred one becomes larger than the one between their most preferred policy and the middle one (thus the movement of the middle policy would correspond to concave utility in our model). That is, if one were to analyze the issues analyzed in this paper by changing the metric in the policy space, then different voters are associated with different metrics over the policy space (Eguia (2010a,b) raises this point. He also points out that the use of concave utility in the Hotelling-Downs framework is warranted only in limited cases. See also Azrieli (2011).) We believe the way we modify the Hotelling-Downs framework is more tractable and directly to the point than the alternative one.

[5]The following is a main excerpt from the act. "No State, territory, or possession of the United States, or Indian tribe, shall be required to give effect to any public act, record, or judicial proceeding of any other State, territory, possession, or tribe respecting a relationship between persons of the same sex that is treated as a marriage under the laws of such other State, territory, possession, or tribe, or a right or claim arising from such relationship."

[6]For more evidence and discussion on issues other than gay marriage, see Glaeser et al. (2005).

To illustrate the intuition, suppose that voter utility is convex and the voter distribution is polarized, and consider the situation where the two candidates announce the optimal policies for the leftist and the rightist voters, respectively. The leftist candidate experiences gain and loss of votes if she moves her policy closer to the middle.[7] The gain is from the centrist and rightist voters. Since the population of centrist voters is small under polarized distributions, the gain from them is small. For the rightist voters, the utility difference between the leftist and the middle policies is relatively small because their utility function is convex. In our probabilistic voting setting, this implies that few rightist voters change their votes, and hence again the gain for the politician is small. On the other hand, the utility difference for the leftist voters between the leftist and the middle policies are relatively large. Thus many leftist voters change their votes, which constitutes a large loss for the politician. Therefore, overall the leftist candidate prefers staying at the leftist policy to moving toward the middle. This is the basic intuition behind the reason why convex utility may imply policy divergence. On the contrary, voters with concave utility functions care more about policy changes when the policy is far from their bliss points. Thus candidates have incentives to position themselves at the middle, so that they can win reasonably many votes from both sides of the distribution of voters. We generalize this argument to characterize a necessary and sufficient condition for policy convergence and divergence and conduct comparative statics. Although the analysis is more complicated, a similar intuition is behind the general result.

We also introduce a model with more than one policy issue, for example, tax policy and same-sex marriage. In that model, the candidates' equilibrium policies diverge on "convex issues," i.e., issues for which voters' utility function is convex, while they converge on "concave issues," as analogously defined. If voters' utility function is convex on religious or moral issues and concave on economic issues (as we discuss below), then our model predicts policy convergence in economic policies and divergence on moral issues, as observed in American politics (see Glaeser et al. (2005)).

In addition to convex utility functions, polarization of voter preferences plays an important role in our model. Popular media has been reporting polarization of Americans in recent years.[8] Some researchers also find suggestive, if not conclusive, evidence of polarization. McCarty et al. (2006) present evidence that suggests the existence of voter

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[7]Exactly the same argument can be made for the rightist candidate.

[8]For example, Gelman (2008, Figure 3.2) finds that newspapers and magazines have recently increased their use of political catchphrases such as "polarizing, polarized," "red state," and "blue state."

polarization and its recent increase in terms of income. DiMaggio et al. (1996) and Evans (2003) find that voters have not polarized on most issues but they have done so on abortion. Other researchers, such as Fiorina (2006), argue that polarization does not exist, or at least has not increased among the general public over the past few decades, but that politicians have become polarized.[9] We do not take a strong stance on empirical evidence and instead provide theoretical predictions on electoral outcomes given voter distributions, an approach that enables us to understand the implications of voter polarization.

The voting literature on policy divergence is so huge that we do not attempt to present a complete literature review here.[10] Instead, we discuss a small subset of them and highlight their differences from our paper. Palfrey (1984) considers the possibility of a third party candidate; Alesina (1988) studies repeated interactions of policy-motivated candidates; Roemer (1994) investigates policy-motivated candidates in a setting with aggregate uncertainty about the position of the median voter; Osborne and Slivinski (1996) and Besley and Coate (1997) consider citizen-candidate models; Callander (2005) analyzes a model with multiple districts; Aragones and Palfrey (2002), Castanheira et al. (2010), Groseclose (2001), and Kartik and McAfee (2007) allow for differences in the personal qualities of candidates (such as charisma); and Glaeser et al. (2005) consider the abilities of politicians to target political messages toward their core constituents, among others. Compared to these works, the departure of our model from the Hotelling-Downs framework is kept fairly small. For example, we obtain divergent equilibria even without policy motivation of candidates or the possibility of entry by a third party. In addition, our explanation of policy convergence and divergence based on voters' utility function is novel[11] and enables us to obtain insights on the relationship between equilibrium policies and social welfare.

The plan of the paper is as follows. Section 2 introduces the model. Section 3 studies a uni-dimensional policy space. In Subsection 3.1, we consider a special case in which the voter distribution is perfectly polarized. The intuition behind the results in this subsection helps us understand the intuition for our main results in the next subsection. In Subsection 3.2, we consider general voter distributions. We formally analyze how the

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[9]See also Abramowitz (2010).

[10]See, for instance, Persson and Tabellini (2002) for a more comprehensive survey.

[11]While Osborne (1995) advocates research on voting with convex voter utility functions, neither that work nor any subsequent work has found a link between convexity and policy divergence to the best of our knowledge.

degrees of voter polarization and convexity of voters' utility function influence policy positions in equilibrium. We also consider the welfare implications of the convexity of voters' utility function. In Section 4, we consider the case of a multi-dimensional policy space. Section 5 concludes. All the proofs are relegated to the Appendix.

2 Basic Model

There is a one-dimensional policy space, $\mathcal{P} := \{0, \frac{1}{2}, 1\}$, where the interpretation is that policies 0, $\frac{1}{2}$, and 1 correspond to the leftist, the centrist, and the rightist policies, respectively. A continuum of voters are distributed according to a probability mass function $f : \mathcal{P} \to [0,1]$ with $f\left(\frac{1}{2}\right) = c$ and $f(0) = f(1) = \frac{1-c}{2}$ for $c \in [0, \frac{1}{3})$.[12] Parameter c represents the degree of centralization of the voter distribution. Two candidates A and B simultaneously determine their positions, x_A and x_B. Candidate $i = A, B$ obtains a share of votes

$$P(x_i, x_{-i}) = \sum_{x \in \mathcal{P}} \sigma \left(u(|x - x_i|) - u(|x - x_{-i}|) \right) f(x), \tag{1}$$

where x_{-i} is the policy position taken by i's opponent, $u : \mathcal{P} \to \mathbb{R}$ is a decreasing function, i.e. $u(x) < u(x')$ if $x > x'$, and $\sigma : \mathbb{R} \to [0,1]$ is a strictly increasing function that satisfies $\sigma(t) + \sigma(-t) = 1$ and weak concavity for all $t \in [0, \infty)$.[13]

Two commonly used specifications of σ in the literature are a logistic function $\hat{\sigma}^{\lambda}(t) = \frac{1}{1 + e^{-\lambda t}}$ with parameter $\lambda > 0$ (logit model), and a cumulative distribution function of a normal distribution $\tilde{\sigma}^p(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi/p^2}} e^{-p^2 z^2 / 2} dz$ with parameter $p > 0$ (probit model).

The above model (1) is referred to as a probabilistic voting model.[14] Let us provide one "micro foundation" of this model because, while standard in the literature, it will help us interpret the results of this paper.[15] Function u is the voters' deterministic utility

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[12]As we mentioned in the Introduction, we focus on the voter distributions that entail some degree of polarization, by assuming $c < \frac{1}{3}$. It will turn out in Theorem 1 and Proposition 5 Part 1 that if $c \geq \frac{1}{3}$ then policy convergence is the only Nash equilibrium of our model. Divergence can be a Nash equilibrium when $c > \frac{1}{3}$ if we assume that voters at the middle incur smaller cost from suboptimal policies than extreme voters, but we refrain from including this possibility for simplicity.

[13]Our main result can be extended to cases with a weaker assumption that σ is nondecreasing, but the statement of the result is more complicated with that assumption. See Appendix A.8 for detail.

[14]See Ledyard (1984) for a number of basic results on probabilistic voting. See also Hinich (1977) and Hinich et al. (1972, 1973).

[15]Other interpretations are given in the literature (see, for example, Persson and Tabellini (2002)). For example, voters may not optimize but vote randomly, or the randomness may reflect subjective beliefs on the part of candidates. The interpretation we give here seems to be the most consistent with the

function which is assumed to be homogeneous across them. The voters are subject to independently and identically distributed random shocks that affect their relative utility between the two candidates. Specifically, the utility difference of a voter with bliss point x between the case when the elected candidate is i and the case when the elected candidate is $j \neq i$ is written as $u(|x - x_i|) - u(|x - x_j|) + \xi$, where the term $u(|x - x_i|) - u(|x - x_j|)$ reflects the utility difference from the policies implemented, and ξ is a random shock on utility difference between candidates i and j for that voter. Each voter is assumed to vote for the candidate who generates more overall utility to her if she is elected, knowing her own realized values of the random term.[16,17] If the random utility shock ξ follows the cumulative distribution function σ, then this voter votes for candidate i with probability $\sigma(u(|x - x_i|) - u(|x - x_j|))$. Our formulation implicitly assumes that no voters abstain, so that $P(x_A, x_B) + P(x_B, x_A) = 1$ for any pair of x_A and x_B.

The logit and probit specifications correspond to special cases of (1) where the random shock term ξ is a difference of two i.i.d. random shocks ξ_A and ξ_B (each of which could be interpreted as random utility shock for each candidate) that follow the extreme value distribution and normal distribution, respectively, independently and identically across voters. The parameters λ and p measure how strongly voters respond to differences in policy positions. Intuitively, the larger they are, the more the voters care about the policy positions.[18] In these models, the voting behavior approximates a deterministic voting behavior of the Hotelling-Downs model as λ and p approach infinity, while the voting behavior becomes completely random as they approach zero. While the logit and probit specifications are used in the literature extensively[19] and we will repeatedly mention them for concreteness, all our results hold more generally under specification (1). In fact, this specification does not even assume that the random shock term ξ is composed of a difference of two i.i.d. shocks for the two candidates. Thus, for instance, if the quality of one candidate is less uncertain than the other (because, say, the former is an incumbent and the latter is a challenger), then the random utility shocks for these two candidates

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standard rational choice framework.

[16]When both candidates generate the same overall utility to a voter, each candidate is chosen by this voter with some arbitrary probability. The specification does not affect the analysis since such an event happens with probability zero.

[17]Since there are only two candidates, voting for the candidate with the higher overall utility is a weakly dominant strategy for each voter. See Myerson and Weber (1993) and Fey (1997) for issues related to voters' strategic behavior when there are more than two candidates.

[18]In the current context, one could interpret λ or p as the "salience" or relative importance of the political issue in consideration, compared to the idiosyncratic utility for each voter.

[19]See, for example, Anderson et al. (1992) and Yang (1995).

may be different. Our model is general enough to accommodate such a situation.[20]

Without loss of generality, we normalize the utility function by setting $u(0) = 1$ and $u(1) = 0$, and set $u\left(\frac{1}{2}\right) = \frac{1}{2} - v$ where $v \in (-\frac{1}{2}, \frac{1}{2})$. Parameter v denotes the degree of convexity of the utility function. Hence, we say that u is **convex** if $v \geq 0$ and u is **concave** if $v \leq 0$. We say that u is strictly convex if $v > 0$ and u is strictly concave if $v < 0$.

There are only three points in the policy space in our model. While it may be tempting to assume that voters are distributed over a continuous policy space, our three-point model has at least two advantages over such a model. First, our model is simple and very tractable. With a continuous policy space, by contrast, even the existence of Nash equilibrium is not guaranteed. Second, our model allows us to unambiguously order all possible voter utility functions with respect to their convexity/concavity by a single parameter v. With a continuous policy space, by contrast, utility functions cannot necessarily be ordered by their convexity/concavity.[21] Perhaps motivated by similar considerations, other studies such as Aragones and Postlewaite (2002) and Gul and Pesendorfer (2009) also use models with three policy outcomes.

Given a profile of positions (x_A, x_B) chosen by the candidates, let $w(x_A, x_B)$ be the "winner" of the election: Formally, let $w(x_A, x_B)$ be i if $P(x_i, x_{-i}) > \frac{1}{2}$, and A and B each with probability $\frac{1}{2}$ if $P(x_A, x_B) = P(x_B, x_A) = \frac{1}{2}$.[22] Each candidate $i = A, B$ has a payoff function that depends largely on whether she is elected but also slightly on the realized policy:

$$U_i(x_A, x_B) = a_i \cdot \mathbb{I}_{\{i=w(x_A,x_B)\}} + \epsilon b_i(|x_{w(x_A,x_B)} - \bar{x}_i|), \tag{2}$$

where a_i is a positive constant that captures the intensity of preferences for being elected, ϵ is a nonnegative constant, and $b_i(\cdot)$ is a decreasing function corresponding to policy preferences, whose argument is the distance between the realized policy and i's bliss

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[20]As an extreme case, if the quality of one candidate is known so that the random utility shock for her is always zero and the other candidate's quality has random shocks following the distribution function σ, then the voting behavior follows the specification represented by (1) associated with function σ.

[21]A possible approach for analyzing a model with a continuous policy space may be to consider a parameterized family of simple voter distributions and utility functions. However, even if a Nash equilibrium exists in such a model, characterizing the equilibria (which we are able to obtain in our discrete model) appears to be difficult. It is relatively straightforward to check local optimality of an announced policy by taking first order conditions, but that is not sufficient for establishing global optimality.

[22]Note that the function $w(\cdot)$ is deterministic unless $P(x_A, x_B) = P(x_B, x_A) = \frac{1}{2}$ even though the model involves probabilistic voting. This is not a contradiction because there is a continuum of voters, which approximates a situation where there are a large number of voters.

point, denoted by $\bar{x}_i$.[23] We assume that $\bar{x}_A = 0$ and $\bar{x}_B = 1$. We assume that, for each i, $\epsilon\left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)\right) < a_i$. This assumption holds if b_i is concave (i.e. $b_i(\frac{1}{2}) \geq \frac{b_i(0)+b_i(1)}{2}$) or if ϵ is sufficiently small. As we formally state in Proposition 4, this assumption implies that candidates primarily care about the vote share, and if two policy choices give candidate A (resp. candidate B) the same probability of winning the election, she prefers the left (resp. the right) policy. In the remainder of the paper, we assume $\epsilon > 0$ unless stated otherwise.

As will become clear, *this policy preference term is not needed to obtain divergent equilibria.* The sole reason to introduce this term is to rule out the possibility of an uninteresting multiplicity of equilibria as we will state in footnote 24. We will show that even though there are multiple equilibria when $\epsilon = 0$, there is no convergent equilibrium $(\frac{1}{2}, \frac{1}{2})$ when divergence is an equilibrium (except for a knife-edge case).

A profile of mixed strategies is a Nash equilibrium if the strategy of each candidate maximizes her expected utility given the strategy of the other candidate.

Following a common approach in the literature (see Banks and Duggan (2005), for instance), we define (utilitarian) social welfare of a policy x by

$$W(x) = \sum_{x' \in \mathcal{P}} u(|x' - x|)f(x').$$

We say that a (mixed) strategy profile is **welfare-maximizing** if for all (x_A, x_B) that are realized with positive probability under that strategy profile, $P(x_i, x_{-i}) \geq \frac{1}{2}$ implies $W(x_i) \geq W(x')$ for all $x' \in \mathcal{P}$. That is, every policy position that wins the election with positive probability maximizes social welfare.

3 Policy Divergence and Policy Convergence

3.1 Illustrative Example: Perfectly Polarized Distribution

This subsection is devoted to the analysis in a particularly simple environment. To this end, we consider a **perfectly polarized** distribution, i.e. $f\left(\frac{1}{2}\right) = c = 0$. That is, f is a perfectly polarized distribution if it has point masses on 0 and 1, each of which has a weight of $\frac{1}{2}$. A perfectly polarized distribution emerges in a political situation where one

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[23]$\mathbb{I}_{i=w(x_A, x_B)}$ is an indicator function, which takes one if candidate i is the winner of the election and zero otherwise.

half of the voters share one bliss point, and the other half share the other bliss point. We do not regard perfect polarization as realistic. Rather, the results in this subsection help understand the intuition for our main results presented in the next subsection.

Proposition 1. *Suppose the voter distribution is perfectly polarized.*

1. If the voters' utility function is convex, then $(0, 1)$ is the unique Nash equilibrium.[24]

2. Otherwise, $(\frac{1}{2}, \frac{1}{2})$ is the unique Nash equilibrium.

Taken together, these two parts of the proposition show the uniqueness of Nash equilibrium for any utility function of voters.

We offer intuition of Proposition 1. Suppose that candidates A and B are at 0 and 1, respectively, and consider the incentive of candidate A. Candidate A experiences a gain and loss by moving from 0 to $\frac{1}{2}$: she receives more votes from the voters at 1, while she loses votes from the voters at 0. If voters have a convex utility function, they care more about policy changes when the proposed policy is close to their bliss points than when it is far. If candidate A moves toward the middle ($\frac{1}{2}$), then the amount of votes she loses from the voters close to her (i.e. the voters at 0) is greater than the amount she gains from the voters far away (i.e. the voters at 1). A symmetric argument holds for candidate B. Thus divergence is an equilibrium when voters have convex preferences. On the contrary, voters with concave utility functions care more about policy changes when the policy is far from their bliss points. Thus candidates have incentives to position at the middle, so that they can win reasonably many votes from both sides of the distribution of voters.

Note that the prediction does not rely on the specification of σ as long as our assumptions are satisfied, so the result is robust with respect to the choice of this function.

Voters' utility function is often assumed to be concave in the literature, and policy convergence has been shown under that assumption (see Banks and Duggan (2005)). Proposition 1 demonstrates the importance of the concavity assumption for such results by showing that both policy convergence and divergence can occur depending on the utility functions. Note that Proposition 1 provides a necessary and sufficient condition for policy convergence, which is uncommon in the literature.

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[24]For the uniqueness of the equilibrium, we utilize the assumption that candidates have preferences over policies ($\epsilon > 0$). The strategy profile $(1, 0)$, for example, is not a Nash equilibrium because candidate A (resp. B) has an incentive to move to policy 0 (resp. 1) as long as $\epsilon > 0$.

Proposition 2. *Suppose the voter distribution is perfectly polarized. Then the (unique) Nash equilibrium is welfare-maximizing.*

As seen in Proposition 1, we may or may not observe policy divergence in equilibrium, depending on voters' utility function. However, Proposition 2 demonstrates that the social optimum is attained in equilibrium, whether or not the divergence occurs.

Note that Proposition 2 enables analysts to evaluate social welfare *without* reference to primitives of the model except voter distributions. This is potentially useful, as analysts can make welfare judgments without much information, such as realized policy positions or utility functions.

In the next subsection, we consider more general voter distributions than the perfectly polarized distribution. It will turn out that many, though not all, of the insights in this section carry over to those general cases.

3.2   Main Results

In this subsection, we investigate how equilibrium policies are affected by voters' utility function, randomness added to it, and polarization of the voter distribution.

Before presenting the main results, we offer two basic results that prove useful in subsequent analysis. First, the following result allows us to focus on pure strategy equilibria without loss of generality.

Proposition 3. *Each candidate uses a pure strategy in any Nash equilibrium.*

Before presenting the second result, recall that we assume $\epsilon(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)) < a_i$ for each i. As we mentioned, this condition holds whenever $\epsilon > 0$ is sufficiently small. This distinguishes our model with most existing ones with policy-motivated candidates, in that effectively we only assume lexicographically weaker preferences for policies than politicians' primary interests in winning the election. In that sense the departure of our setup from the standard Hotelling model is kept fairly small. Indeed, it is clear from the definition that any strict Nash equilibrium in the game with $\epsilon = 0$ is a Nash equilibrium of the game with sufficiently small $\epsilon > 0$, so our requirement of candidates' policy preferences is mild. Even so, interestingly this policy preference term does rule out some equilibria and enables us to obtain a unique prediction under certain circumstances as we will see shortly. In fact, the following proposition shows that Nash equilibria with $\epsilon > 0$ are equivalent to a certain refinement of Nash equilibria of the game with $\epsilon = 0$.

Proposition 4. (x_A^*, x_B^*) *is a Nash equilibrium of the game with $\epsilon > 0$ if and only if it is a Nash equilibrium of the game with $\epsilon = 0$ with the additional property that, for each i, there exists no x_i' such that $P(x_i', x_{-i}^*) = P(x_i^*, x_{-i}^*)$ and $|x_i' - \bar{x}_i| < |x_i^* - \bar{x}_i|$.*

The proposition shows that analyzing a game with $\epsilon > 0$ is equivalent to analyzing a game with $\epsilon = 0$ as long as we analyze a (slight) refinement of Nash equilibrium, in the sense that we focus on Nash equilibria in which each candidate i announces the closest policy to $\bar{x}_i$ among the set of policies that gives the highest vote share to her. In this sense, the model with policy preferences ($\epsilon > 0$) departs from the standard model of purely office-motivated candidates ($\epsilon = 0$) in a fairly small manner. Indeed, all our results except for uniqueness of equilibrium hold also with $\epsilon = 0$. Thus, even if we stick to the model with politicians whose only objective is to maximize vote share, we still obtain policy divergence as a Nash equilibrium under a wide range of environments.

Now we are ready to present our main theorem. The theorem provides a necessary and sufficient condition for the equilibrium to exhibit policy divergence/convergence, in terms of convexity of voter utility and voter polarization. Recall that c denotes the degree of centralization of the voter distribution. For any σ, let

$$\bar{c}(v, \sigma) = \left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)} \right)^{-1}$$

for any $v > 0$, and $\bar{c}(0, \sigma) = 0$.[25] For instance, with the logistic function $\hat{\sigma}^\lambda$, this expression can be written as $\bar{c}(v, \hat{\sigma}^\lambda) = \left(2 + \frac{e^{\frac{1}{2}\lambda} - e^{-\frac{1}{2}\lambda}}{e^{v\lambda} - e^{-v\lambda}} \right)^{-1}$ for any $v > 0$, and $\bar{c}(0, \hat{\sigma}^\lambda) = 0$ (Figure 1 just before Theorem 2 plots $\bar{c}(v, \hat{\sigma}^\lambda)$ versus v for various values of λ).

Theorem 1. *1. If voters' utility function is convex and $c \leq \bar{c}(v, \sigma)$, then $(0, 1)$ is the unique Nash equilibrium.*

2. Otherwise, $(\frac{1}{2}, \frac{1}{2})$ is the unique Nash equilibrium.

The basic intuition of this theorem can be explained in an analogous manner as we did for Proposition 1. Suppose that candidates A and B are at 0 and 1, respectively, and consider the incentive of candidate A. Candidate A experiences a gain and loss by moving from 0 to $\frac{1}{2}$: the gain from obtaining more votes from the voters at $\frac{1}{2}$ and 1, and the loss from obtaining less votes from the voters at 0. If voters' utility function is sufficiently

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[25] $\bar{c}(0, \sigma) = \lim_{v \searrow 0} \bar{c}(v, \sigma)$ by Proposition 5 below.

convex, they care more about policy changes when the proposed policy is close to their bliss points than when it is far.[26] If candidate A moves toward the middle ($\frac{1}{2}$), then the amount of votes she loses from the voters close to her (i.e. the voters at 0) is greater than the amount she gains from the voters far away (i.e. the voters at 1). A symmetric argument holds for candidate B. Thus divergence is an equilibrium when voters' utility function is sufficiently convex. The other cases can be similarly explained.

Further intuition can be explained as follows: Suppose $v > 0$ and that $(0, 1)$ is a Nash equilibrium for some $c = c'$. Then, for any $c < c'$, we should expect it to be a Nash equilibium. Intuitively speaking, this is because, since more voters are at the "extreme positions" (i.e. 0 or 1) when $c < c'$ than when $c = c'$, the convexity of voters' utility function implies (with the logic explained in the previous subsection) more incentive for the candidates to situate themselves at extreme positions.[27] Conversely, if $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium for some $c = c''$, then for any $c > c''$ we should expect it to be a Nash equilibrium. This is because, since more voters are at the "middle points" when $c > c''$ than when $c = c''$, the incentive to take "extreme points" (implied by the convexity of u) decreases, so candidates still want to take the middle position. In other words, there exist thresholds, $c_*(v, \sigma)$ and $c^*(v, \sigma)$, for divergence and convergence, respectively, to be equilibria.

While we have explained the intuition for Theorem 1, note that there is more content in this theorem: (i) there exists a unique Nash equilibrium for each c, (ii) thresholds $c_*(v, \sigma)$ and $c^*(v, \sigma)$ are identical, and (iii) we can analytically solve for the threshold $\bar{c}(v, \sigma)$. (iii) is useful since it enables us to conduct simple comparative statics, as shown in the next proposition.

We will investigate comparative statics of the threshold $\bar{c}(v, \sigma)$ with respect to various parameters of the model. To do so, first we introduce some concepts. Given σ and $\rho > 0$, define the ρ-**transformation** of σ by $\sigma^\rho(t) = \sigma(\rho t)$ for all t.[28] If $\rho > 1$ then a voter's behavior is "less random" under σ^ρ than under σ in the sense that it is closer to the deterministic voting as assumed in the Hotelling (1929) model. We say that the function

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[26]The cases for $v \leq 0$ is standard in the literature. See, for example, Banks and Duggan (2005).

[27]We point out that, given a convex voter utility function, the existence of small enough a value c such that policy divergence is an equilibrium under c is not surprising given Proposition 1. Nontrivial contents of this theorem are explained shortly.

[28]Parameter ρ represents the weight placed on the deterministic utility relative to the random utility shock (which is normalized to one). That is, we change a voter's utility from $u(|x - x_i|) - u(|x - x_j|) + \xi$ to $\rho[u(|x - x_i|) - u(|x - x_j|)] + \xi$ while the distributions of ξ are kept identical in both cases.

σ satisfies the **increasing average-marginal ratio property** if it is differentiable and $\frac{\sigma(t)-\sigma(0)}{t\sigma'(t)}$ is strictly increasing for all $t \geq 0$. To interpret this property, note that $\frac{\sigma(t)-\sigma(0)}{t}$ is the average rate of increase in the probability of voting for the closer candidate between 0 and t, while $\sigma'(t)$ is the marginal rate of increase in the probability of voting for the closer candidate at t. The property requires that the ratio of these two rates be increasing in t. While the condition excludes some functions, many functions that are of interest, such as those used in logit and probit models, satisfy it (we prove this fact in Appendix A.7). With the above definitions, we are ready to present comparative statics results of the threshold $\bar{c}(v, \sigma)$ with respect to various parameters of the model.

Proposition 5.   *1. $\bar{c}(v, \sigma)$ is strictly increasing in v, $\lim_{v \searrow 0} \bar{c}(v, \sigma) = 0$, and $\lim_{v \nearrow \frac{1}{2}} \bar{c}(v, \sigma) = \frac{1}{3}$ for any σ.*

  2. (a) If σ satisfies the increasing average-marginal ratio property, then $\bar{c}(v, \sigma^\rho)$ is strictly decreasing in $\rho > 0$ for any $v > 0$.

   (b) $\bar{c}(v, \sigma) \leq \frac{2v}{4v+1}$ for any v, σ. If σ is differentiable, then $\lim_{\rho \searrow 0} \bar{c}(v, \sigma^\rho) = \frac{2v}{4v+1}$.

   (c) If sequence $\{\sigma_n\}_{n=1}^\infty$ converges pointwise to σ^* where $\sigma^*(t) = 1$ for all $t > 0$, then $\lim_{n \to \infty} \bar{c}(v, \sigma_n) = 0$.

Part 1 of the Proposition shows that a higher degree of convexity of voters' utility function makes a divergent equilibrium possible even when the voter distribution is more concentrated at the median. Moreover, as voters' utility function approaches a linear function (i.e., $v \to 0$), the threshold $\bar{c}(v, \sigma)$ needed for a divergent equilibrium approaches zero, and when the utility function becomes linear, the divergent equilibrium persists only under the perfectly polarized distribution, $c = 0$. As convexity becomes very large $(v \to \frac{1}{2})$, the divergent equilibrium becomes prevalent and the threshold approaches $c = 1/3$, corresponding to the uniform distribution.

Part 2 of the proposition shows that the more strongly voters care about policies relative to idiosyncratic random preferences, the more polarization is needed for divergent equilibria. Furthermore, as the degree of randomness becomes much more significant than voters' deterministic preference, the threshold needed for a divergent equilibrium approaches a limit that depends on the degree of convexity of the preference (v). As the randomness becomes negligible relative to the deterministic preference of the voters, the divergence becomes difficult and the threshold approaches zero. This result makes

intuitive sense: In the limit as ρ approaches infinity, policy divergence is not an equilibrium for any $c > 0$, because a candidate at 0, say, does not lose any votes from voters at 0 by moving to $\frac{1}{2}$, just as in the standard Hotelling-Downs model.

Part 2c allows for a general sequence of functions that converges pointwise to the deterministic case. In particular, consider ρ-transformations of a function σ. Assume that $\lim_{t\to\infty} \sigma(t) = 1$, that is, a voter's choice probability for a candidate approaches one as the utility of the candidate's position becomes infinitely larger than her opponent's.[29] Then, since $\lim_{\rho\to\infty} \sigma^\rho(t) = \lim_{\rho\to\infty} \sigma(\rho t) = 1$ for any $t > 0$, Part 2c implies that $\lim_{\rho\to\infty} \bar{c}(v, \sigma^\rho) = 0$. In words, the threshold converges to zero in the class of ρ-transformations as ρ becomes infinitely large.

Note that, for any σ, policy divergence occurs under voter distributions that have centralizations close to (but less than) $\frac{1}{3}$ as long as the utility function is sufficiently convex, while even a very large degree of randomness does not necessarily imply policy divergence under such a voter distribution. This analysis shows that sufficient convexity, rather than sufficient randomness, is essential for divergence in equilibrium: Note that only a very small amount of randomness is actually needed. This fact might appear contradictory to the "median voter theorem" as predicted by the standard Hotelling-Downs model, where there is no randomness in voters' utility. However there is no inconsistency. As we have seen in Part 2c, once we fix the value of $v < \frac{1}{2}$, $\bar{c}(v, \sigma_n)$ approaches 0 as the randomness vanishes ($\sigma_n \to \sigma^*$).

We plot in Figure 1 the value of $\bar{c}(v, \sigma)$ with respect to v under the logit specification (i.e. $\sigma = \hat{\sigma}^\lambda$), for four values of λ: 0.0001, 2, 5, 10, and 15. The graph illustrates that more convexity and randomness allow policy divergence under distributions with more centralization. Notice that, for any fixed v, $\bar{c}(v, \hat{\sigma}^\lambda)$ approaches zero as λ increases.

The next result shows that the divergent equilibrium is welfare-maximizing.[30]

Theorem 2. *If $(0, 1)$ is a Nash equilibrium, then it is welfare-maximizing.*

In Proposition 2 we showed that a Nash equilibrium, whatever it is, is welfare-

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[29]This property is regularly assumed in discrete choice theory. For instance, the functions used in logit and probit models for any specification of the parameter satisfy this condition.

[30]Recall that the social welfare used in this paper is utilitarian, that is, the average of voters' utilities. While this may not be the only reasonable measure of welfare, we use it to compare our result to those in the existing literature using this concept (such as Banks and Duggan 2005).
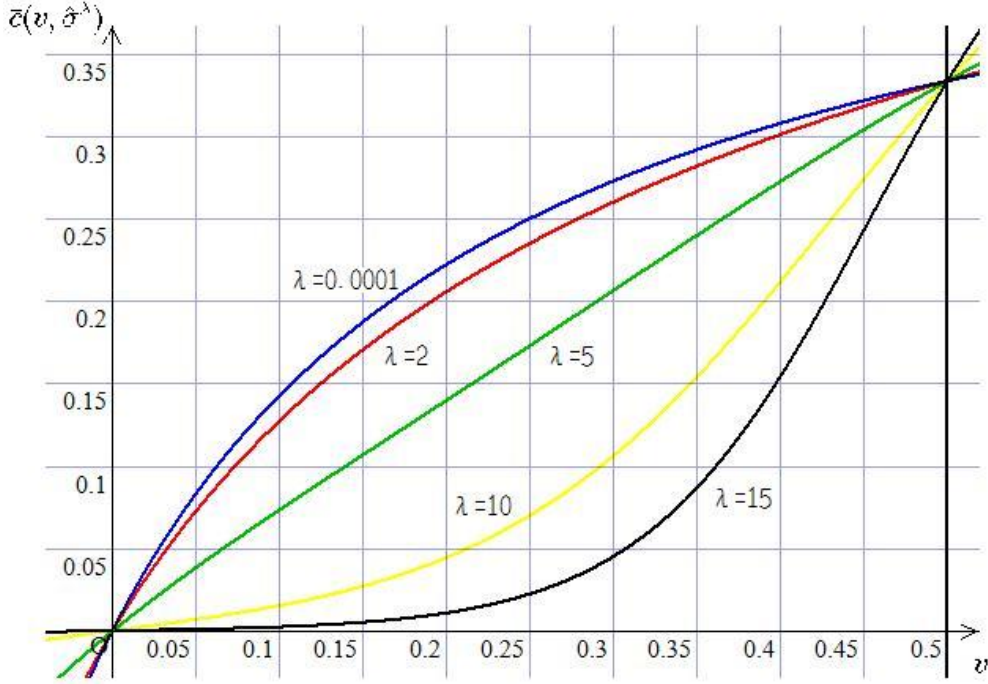
Figure 1: Relationship between v and $\bar{c}(v, \hat{\sigma}^\lambda)$ for different values of λ.

maximizing, when the voter distribution is perfectly polarized. On the other hand, the above theorem applies only when the divergent policy profile is a Nash equilibrium. Actually, the converse of the theorem is not true: Even if $(0, 1)$ is welfare-maximizing, it is not necessarily a Nash equilibrium. Also, in such a case the only equilibrium is $(\frac{1}{2}, \frac{1}{2})$. Hence, in our model with convex utility function, *the convergent equilibrium is not necessarily welfare-maximizing.*[31] This conclusion contrasts with the standard results in the literature (see, for example, Banks and Duggan (2005)) that the convergent equilibrium is welfare-maximizing.

The key to Theorem 2 is that the voting probability is sensitive to the utility difference when the latter is small in absolute term (as σ is weakly concave in it). This implies that when the policy profile is $(0, 1)$, candidate A can increase the votes from the voters at $\frac{1}{2}$ substantially by moving to $\frac{1}{2}$, while these voters experience only a mild increase in payoffs. Therefore if $(0, 1)$ is an equilibrium, then A's gain in votes from moving to $\frac{1}{2}$ is smaller than her loss, so the welfare gain for voters is outweighed by the loss as well, implying that $(0, 1)$ is also welfare-maximizing. An analogous argument implies that A sometimes gains by moving to $\frac{1}{2}$ even if that decreases the welfare, explaining why there exist cases

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[31]An example is as follows: Consider the logit case with $\lambda = 1$, $c = 0.248$, and $v = \frac{1}{4}$. Then, it can be shown that $c = 0.248 > 0.246 \cdots = \bar{c}(\frac{1}{4}, 1)$, so that $(\frac{1}{2}, \frac{1}{2})$ is the unique Nash equilibrium, while it is not welfare-maximizing, since $W(0) = W(1) = 0.438 > 0.436 = W(\frac{1}{2})$.

in which a convergent equilibrium is not welfare-maximizing. Being consistent with this intuition, when voters are perfectly polarized ($c = 0$) the convergent equilibrium is always welfare-maximizing (Proposition 2).

4   Multi-Dimensional Policy Space

The primary purpose of this paper is to study how positions of political candidates are determined in a strategic situation and how positions are related to the nature of the political issues. In this section we propose a model in which there is more than one policy issue. We will see that political candidates diverge on some issues and converge on others in equilibrium, and that issues with divergent policy positions are precisely those on which voters have a convex utility function.

A continuum of voters are distributed on $\mathcal{P} := \{0, \frac{1}{2}, 1\}^n$ according to a probability mass function f on $\mathcal{P}$. The interpretation is that each dimension of the policy space corresponds to one policy issue. Two candidates A and B with bliss points $(\bar{x}_A, \bar{x}_B) = ((0, \ldots, 0), (1, \ldots, 1))$ simultaneously determine their positions, $x \in \mathcal{P}$ and $y \in \mathcal{P}$. The payoff of each candidate i is given by equation (2) as before, except that the vote share function (1) is replaced by its multi-dimensional generalization,

$$
P_i(x, y) = \sum_{x' \in \mathcal{P}} \sigma \left(\sum_{k=1}^n \delta_k u_k(|x'_k - x_k|) - \sum_{k=1}^n \delta_k u_k(|x'_k - y_k|) \right) f(x')
$$

where $\sum_{k=1}^n \delta_k u_k(\cdot)$ represents voters' utility function (and, as before, $\sigma : \mathbb{R} \to [0, 1]$ is a strictly increasing function that satisfies $\sigma(t) + \sigma(-t) = 1$ for any $t \in (-\infty, \infty)$). Implicit in this definition is the assumption that voters' utility function is additive across different policy issues. However, *we let the function σ apply to the sum of these terms,* as opposed to taking the sum of σ's. This means that different issues are not considered to be completely independent. For each k, we assume that u_k is a decreasing function satisfying $u_k(0) = 1$ and $u_k(1) = 0$, and $\delta_k > 0$. Parameter δ_k represents the relative importance of the k'th policy issue for voters.

To obtain a sharp prediction, we assume that the distribution of voters $f : \mathcal{P} \to [0, 1]$ is perfectly polarized, that is, $\text{supp}(f) \subseteq \{0, 1\}^n$ and $f(x) = f(x')$ for all $x, x' \in \mathcal{P}$ with $x'_k = 1 - x_k$ for all $k = 1, \ldots, n$.

The concept of a perfectly polarized distribution is a generalization of the correspond-

ing notion in Subsection 3.1. The class of perfectly polarized distributions subsumes as a special case a distribution that assigns a mass of $\frac{1}{2}$ to $(0, \ldots, 0)$ and a mass of $\frac{1}{2}$ to $(1, \ldots, 1)$. Also included in this class is a distribution where each vertex of the n-dimensional unit cube $\{0, 1\}^n$ has an identical weight and all other points have weight zero. However, the notion of perfectly polarized distribution is more general: For example, in the 2-dimensional policy space, the distribution in which fraction $\frac{1}{3}$ of the voters are situated at $(0, 0)$ and $(1, 1)$ each, and $\frac{1}{6}$ at $(0, 1)$ and $(1, 0)$ each, is a perfectly polarized distribution.

We first provide a characterization of a Nash equilibrium, and then give a welfare analysis.

Theorem 3. *Suppose the voter distribution is perfectly polarized. There exists a unique Nash equilibrium, $(x_A^*, x_B^*) = ((x_{A1}^*, \ldots, x_{An}^*), (x_{B1}^*, \ldots, x_{Bn}^*))$, which is given by*

$$
(x_{Ak}^*, x_{Bk}^*) = \begin{cases} (0, 1) & \text{if } u_k \text{ is convex,} \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{otherwise.} \end{cases}
$$

The result shows that candidates' positions have a clear dichotomy. More specifically, candidates diverge on "convex issues," that is, issues for which voters' utility function is convex, while they converge on "concave issues." The case where $n = 1$ corresponds to Proposition 1.

As in Subsection 3.1, we can show that a Nash equilibrium is welfare-maximizing whether or not it is convergent. We follow essentially the same analysis as before. Extending the definition before, social welfare of a policy x is

$$
W(x) = \sum_{x' \in \mathcal{P}} \sum_{k=1}^{n} \delta_k u_k(|x_k' - x_k|) f(x'),
$$

where $x = (x_1, \ldots, x_n)$ and $x' = (x_1', \ldots, x_n')$.

Given this definition of $W(x)$, we say that a (mixed) strategy profile is welfare-maximizing if for all (x_A, x_B) that realizes with positive probability under that strategy profile, $P(x_i, x_{-i}) \geq \frac{1}{2}$ implies $W(x_i) \geq W(x')$ for all $x' \in \mathcal{P}$. That is, every policy position that wins the election with positive probability maximizes social welfare.

Proposition 6. *Suppose the voter distribution is perfectly polarized. Then the (unique) Nash equilibrium is welfare-maximizing.*

The proof is omitted, as it is a relatively easy adaptation of the proof of Proposition 2.

We have seen in Proposition 2 that the unique Nash equilibrium is welfare-maximizing in the model with a uni-dimensional policy space. The above proposition generalizes this result: The unique Nash equilibrium is welfare-maximizing even in a multi-dimensional policy space.

Finally, by an inspection of the proof of Theorem 3, it can be seen that the conclusions of this section hold for voter distributions that are not perfectly polarized, as long as u_k for each k is either strictly convex or strictly concave and the voter distribution is close enough to a perfectly polarized distribution.[32]

5 Concluding Remarks

We considered a probabilistic voting model with utility functions that are not necessarily concave. As we have discussed in the Introduction, convexity seems a natural assumption when the relevant policy issues involve moral or religious contents. In our model, when voters are more polarized than in the uniform distribution, we showed that there is a convexity threshold of voters' utility function (i) below which policy convergence is a unique equilibrium, and (ii) above which policy divergence is a unique equilibrium. As voters become more polarized, divergent policies prevail in equilibrium for a wider range of voter utility functions. Moreover, social welfare is maximized in each divergent equilibrium, but not necessarily in every convergent equilibrium. When there is more than one policy issue and the voter distribution is perfectly polarized, the candidates' equilibrium policies diverge on issues for which utility functions are convex and converge on issues for which utility functions are concave.

In this paper, we assumed that voting is not costly and no voters abstain. While this assumption is widely adopted in many models including the original ones by Hotelling and Downs (thus making our results comparable to those in the literature), certainly it is restrictive. However, our results hold more generally. Kamada and Kojima (2013) prove an equivalence result between the probabilistic voting model without abstention

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[32]More specifically, we say that a voter distribution f is η-close to a perfectly polarized distribution if $f(x) = f(x')$ for all $x, x' \in \mathcal{P}$ and $f(x) < \eta$ for every $x \notin \{0,1\}^n$. For any voter utility function such that u_k for each k is either strictly convex or strictly concave, the conclusions of this section hold for any voter distribution that is η-close to a perfectly polarized distribution with any sufficiently small η.

and a costly voting model in which voters abstain when a randomly drawn cost of voting exceeds the benefit from voting. More specifically, in two-candidate elections, given any probabilistic voting model, there exists a costly voting model that generates winning probabilities identical to those in the former model for any policy announcements, and vice versa. Thus all the results established in this paper, except for those about social welfare, hold in a corresponding costly voting model that allows for abstention as well, providing robustness of our main finding to model specifications.

Let us conclude the paper by suggesting several future research directions. First, it may be interesting to empirically identify on which policy issues voters have convex and concave utility functions, respectively. The second issue is a generalization to policy spaces with more than three points. As illustrated in Section 2, our three-point model is advantageous because it is tractable and allows us to unambiguously define the degree of convexity of any voter function. Still, extending our results to more general cases may prove useful, for example, to understand the effect of convexity or polarization on the degree of policy divergence. This topic is left for future research: we conjecture that the basic logic underlying policy divergence would extend.[33] Finally, investigating the implications of various utility functions would be a fruitful approach more generally. Van Weelden (2013) shows that sufficiently concave utility functions can lead to policy divergence in a dynamic setting, and Van Weelden (2011) investigates relationship between policy divergence and the degree of concavity of voter utility functions. Convex utility functions have been usefully employed to study ambiguity in elections (Shepsle, 1972; Aragones and Postlewaite, 2002). More recently, studies have shown that convex utility functions imply qualitatively different results from concave utility functions in other contexts, too. For example, Kamada and Kandori (2009) consider a model of a dynamic process in which candidates revise their policies over time until the time of election and show that if candidates have convex utility functions then policies diverge more than in the case with concave utility functions. We expect more studies will blossom from this paper and the idea of convex utility functions more generally.

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[33]Formally generalizing our results is difficult, however. In continuous policy space, for instance, even the existence of a Nash equilibrium is not guaranteed. Even if one exists, a characterization of the equilibria (which we obtain in our model) is nontrivial. It is relatively easy to check local optimality of an announced position by taking first order conditions, but that is not sufficient for global optimality.

A Appendix

Propositions 1 and 2 are proven in Sections A.3 and A.5, respectively. We begin by proving Proposition 3.

A.1 Proof of Proposition 3

Proof. Let $\Delta(\mathcal{P})$ be the set of a candidate's mixed strategies, and for candidate i and any $\alpha_i, \alpha_{-i} \in \Delta(\mathcal{P})$, let

$$U_i(\alpha_i, \alpha_{-i}) := \sum_{(x_i, x_{-i}) \in \mathcal{P}^2} \alpha_i(x_i) \cdot \alpha_{-i}(x_{-i}) \cdot U_i(x_i, x_{-i}),$$

where α_{-i} is the mixed strategy of the candidate different from i. A profile of mixed strategies (α_A, α_B) is a Nash equilibrium if $\alpha_i \in \arg\max_{\alpha_i' \in \Delta(\mathcal{P})} U_i(\alpha_i', \alpha_{-i})$ for each $i = A, B$.

Consider a Nash equilibrium (α_A, α_B). We first show that $\alpha_A(1) = 0$. First note that, by symmetry, A's winning probability is the same when she plays a pure strategy $x_A = 0$ as when she plays a pure strategy $x_A = 1$. Suppose first that the probability that A wins the election at $x_A = 1$ is zero. By symmetry, this occurs only when $\alpha_B(\frac{1}{2}) = 1$ and $P(1, \frac{1}{2}) < \frac{1}{2}$. In this case, by playing $x_A = 1$, A's winning probability is 0, while the realized policy is $\frac{1}{2}$ with probability 1. But by playing $x_A = \frac{1}{2}$, A's winning probability is $\frac{1}{2}(> 0)$, while the realized policy is, again, $\frac{1}{2}$ with probability 1. Thus in this case A cannot put a positive probability on $x_A = 1$ in a Nash equilibrium.

Consider next the other case, i.e. the case in which the probability that A wins the election at $x_A = 1$ is positive. Notice again that, by symmetry, the winning probability is the same when A plays $x_A = 0$ as when she plays $x_A = 1$. Since whenever she wins the realized policy is closer to her bliss point when $x_A = 0$ than when $x_A = 1$, pure strategy $x_A = 0$ gives a strictly higher payoff to candidate A than pure strategy $x_A = 1$ does. Thus, again in this case, candidate A puts probability zero on $x_A = 1$ in any Nash equilibrium. Therefore, we conclude that $\alpha_A(1) = 0$. A symmetric argument shows that $\alpha_B(0) = 0$.

In order to prove the proposition, now consider two cases. Suppose first that $P(0, \frac{1}{2}) \geq \frac{1}{2}$. Given any realized action of B, when $x_A = 0$, A's winning probability is strictly positive and weakly larger than when $x_A = \frac{1}{2}$. Also, whenever A wins the election, the realized

policy is strictly closer to A's bliss point when $x_A = 0$ than when $x_A = \frac{1}{2}$, and whenever A loses, it is the same when $x_A = 0$ as when $x_A = \frac{1}{2}$. Hence under this assumption candidate A takes a pure strategy $x_A = 0$ in the best response.

Next, suppose that $P(0, \frac{1}{2}) < \frac{1}{2}$. Then, candidate A's expected payoff from $x_A = 0$ is:

$$\frac{\alpha_B(1)}{2} a_A + \epsilon \left(\frac{\alpha_B(1)}{2} b_A(0) + (1 - \alpha_B(1)) b_A(\frac{1}{2}) + \frac{\alpha_B(1)}{2} b_A(1) \right).$$

On the other hand, her expected payoff from $x_A = \frac{1}{2}$ is:

$$\frac{1 + \alpha_B(1)}{2} a_A + \epsilon b_A(\frac{1}{2}).$$

We show that the latter is strictly larger than the former. To see this, subtract the former from the latter:

$$\left(\frac{1 + \alpha_B(1)}{2} a_A + \epsilon b_A(\frac{1}{2}) \right) - \left(\frac{\alpha_B(1)}{2} a_A + \epsilon \left(\frac{\alpha_B(1)}{2} b_A(0) + (1 - \alpha_B(1)) b_A(\frac{1}{2}) + \frac{\alpha_B(1)}{2} b_A(1) \right) \right)$$
$$= \frac{1}{2} \left(a_A - \alpha_B(1) \cdot \epsilon \left(b_A(0) - 2 b_A(\frac{1}{2}) + b_A(1) \right) \right). \tag{3}$$

Expression (3) is obviously strictly positive if $b_A(0) - 2 b_A(\frac{1}{2}) + b_A(1)$ is negative. Otherwise, (3) is the smallest if $\alpha_B(1) = 1$, in which case it is strictly positive because of the assumption that $\epsilon \left(b_A(0) - 2 b_A(\frac{1}{2}) + b_A(1) \right) < a_A$. This shows that $x_A = \frac{1}{2}$ is the unique best response. Hence candidate A takes a pure strategy in a Nash equilibrium.

A symmetric argument shows that candidate B uses a pure strategy in any best response, completing the proof. $\square$

A.2 Proof of Proposition 4

Proof. We show the "Only if" direction first and then the "If" direction.

"Only if" direction. Let (x_A^*, x_B^*) be a pure strategy Nash equilibrium of the game with $\epsilon > 0$ (by Proposition 3, we can focus on pure strategy Nash equilibria without loss of generality). We begin by showing that each candidate obtains the vote share of $\frac{1}{2}$. Suppose the contrary, i.e. that some candidate i gets a vote share strictly smaller than $\frac{1}{2}$. Then, by deviating from x_i^* to $x_i = x_{-i}^*$, candidate i can strictly increase the winning probability, while this deviation does not change the realized policy (namely x_{-i}^*). This

contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium. Hence in (x_A^*, x_B^*), each candidate gets the vote share of $\frac{1}{2}$. Thus, in particular, the winninng probability in (x_A^*, x_B^*) is $\frac{1}{2}$.

From the proof of Proposition 3, $x_A^* \neq 1$ and $x_B^* \neq 0$. Hence there are four cases: (i) $(x_A^*, x_B^*) = (0, \frac{1}{2})$, (ii) $(x_A^*, x_B^*) = (0, 1)$, (iii) $(x_A^*, x_B^*) = (\frac{1}{2}, \frac{1}{2})$, and (iv) $(x_A^*, x_B^*) = (\frac{1}{2}, 1)$.

In each case, we will suppose that candidate A has an incentive to deviate from (x_A^*, x_B^*) in the game with $\epsilon = 0$, and derive contradictions. By symmetry this is sufficient to show that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon = 0$.

First, consider case (i). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = \frac{1}{2}$ has to give a strictly higher winning probability than x_A^*. But it would give A the winning probability of $\frac{1}{2}$ by symmetry, which is the same as when she takes x_A^*, a contradiction.

Second, consider case (ii). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = \frac{1}{2}$ has to give a strictly higher winning probability than x_A^* (1 instead of $\frac{1}{2}$). But then by assumption $\epsilon \left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1) \right) < a_i$, we have $a_i + \epsilon b_i(\frac{1}{2}) > \frac{a_i}{2} + \frac{\epsilon}{2} \left(b_i(0) + b_i(1) \right)$, which implies that in the game with $\epsilon > 0$, $x_A = \frac{1}{2}$ gives a higher payoff to candidate A than x_A^* does. This contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

Third, consider case (iii). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = 0$ has to give a strictly higher winning probability than x_A^*. But if it were the case, then in the game with $\epsilon > 0$, the deviation from x_A^* to 0 gives candidate A a higher winning probability (1 instead of $\frac{1}{2}$) as well as the realized policy closer to her bliss point (0 instead of $\frac{1}{2}$). Thus candidate A would be better off by taking 0 instead of x_A^*, which contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

Finally, consider case (iv). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = 0$ has to give a strictly higher winning probability than x_A^*. But it would give A the winning probability of $\frac{1}{2}$ by symmetry, which is the same as when she takes x_A^*, a contradiction.

To conclude, we have shown that whenever (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$, it is also a Nash equilibrium in the game with $\epsilon = 0$.

Now we verify that in (x_A^*, x_B^*) candidate A does not have another choice x_A' such that $|x_A' - \bar{x}_A| < |x_A^* - \bar{x}_A|$ that gives her the same vote share. If such x_i' exists, then it is immediate that the payoff from x_A^* is strictly less than the one from x_A' in the game with $\epsilon > 0$, since x_A' is closer to A's bliss point than x_A^*. This contradicts the assumption that

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(x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

"If" direction. Let (x_A^*, x_B^*) be a Nash equilibrium of the game with $\epsilon = 0$ with the property that, for each $i = A, B$, candidate i has no other choice of policy x_i' such that $P(x_i', x_{-i}^*) = P(x_i^*, x_{-i}^*)$ and $|x_i' - \bar{x}_i| < |x_i^* - \bar{x}_i|$. This implies that the vote share at x_i^* is $\frac{1}{2}$, and there is no other choice x_i' that gives strictly higher vote share. Suppose that (x_A^*, x_B^*) is not a Nash equilibrium of the game with $\epsilon > 0$. In the game with $\epsilon > 0$, the expected payoff at x_i^* is $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$. The expected payoff at another choice x_i' is $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$ if x_i' gives the vote share of $\frac{1}{2}$, and it is $\epsilon b_i(|x_{-i}^* - \bar{x}_i|)$ if x_i' gives the vote share strictly less than $\frac{1}{2}$.

In the former case, to compare

$$\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$$

and

$$\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right),$$

note that x_i^* and x_i' give the same vote share. This implies, by assumption, that x_i^* is weakly closer to $\bar{x}_i$ than x_i' is. This implies that $b_i(|x_i^* - \bar{x}_i|) \geq b_i(|x_i' - \bar{x}_i|)$. Hence

$$\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right) \geq \frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right). \tag{4}$$

In the latter case, first note that $x_A^* \neq 1, x_B^* \neq 0$ by the proof of Proposition 3, so $|x_i^* - \bar{x}_i| \leq |x_{-i}^* - \bar{x}_i|$. Therefore

$$\epsilon b_i(|x_{-i}^* - \bar{x}_i|) \leq \frac{\epsilon}{2}(b_i(|x_A^* - \bar{x}_i|) + b_i(|x_B^* - \bar{x}_i|)$$
$$< \frac{a_i}{2} + \frac{\epsilon}{2}(b_i(|x_A^* - \bar{x}_i|) + b_i(|x_B^* - \bar{x}_i|). \tag{5}$$

Conclusions (4) and (5) of these two cases show that candidate i does not have an incentive to deviate from (x_A^*, x_B^*) in the game with $\epsilon > 0$. This completes the proof.

$\square$

A.3 Proofs of Proposition 1 and Theorem 1

Since Proposition 1 is a special case of Theorem 1 when $c = 0$, we prove Theorem 1.

Proof. First, we show that if there exists a Nash equilibrium, then it must be unique. To see this, let (x_A^*, x_B^*) and (x_A^{**}, x_B^{**}) be Nash equilibria. Since each candidate obtains the vote share of $\frac{1}{2}$ in any equilibrium as shown in the proof of Proposition 4, we have

$$\frac{1}{2} = P(x_A^*, x_B^*) \geq P(x_A^{**}, x_B^*) = 1 - P(x_B^*, x_A^{**}) \geq 1 - P(x_B^{**}, x_A^{**}) = P(x_A^{**}, x_B^{**}) = \frac{1}{2}.$$

Hence, it must be the case that $P(x_A^*, x_B^*) = P(x_A^{**}, x_B^*)$. This equation together with the condition that (x_A^*, x_B^*) is a Nash equilibrium implies that $|x_A^* - \bar{x}_A| \leq |x_A^{**} - \bar{x}_A|$. By a symmetric argument, we obtain $|x_A^* - \bar{x}_A| \geq |x_A^{**} - \bar{x}_A|$ Hence, we have $|x_A^* - \bar{x}_A| = |x_A^{**} - \bar{x}_A|$. Since $\bar{x}_A = 0$, we have $x_A^* = x_A^{**}$. We can apply an analogous argument to show that $x_B^* = x_B^{**}$. Therefore, we conclude that if there exists a Nash equilibrium, then it must be unique.

Part 1. We will show that $(x_A^*, x_B^*) = (0, 1)$ is a Nash equilibrium (and hence the unique Nash equilibrium) if $v \geq 0$ and $c \leq \bar{c}(v, \lambda)$.

By Proposition 4, the assumption $(\bar{x}_A, \bar{x}_B) = (0, 1)$, and symmetry, $(0, 1)$ is a Nash equilibrium if and only if the vote share of candidate B is at most $\frac{1}{2}$ when she chooses position $\frac{1}{2}$ while candidate A chooses 0. This condition is equivalent to:

$$P(\frac{1}{2}, 0) \leq \frac{1}{2}$$
$$\iff \frac{1-c}{2} \cdot \sigma(u(\frac{1}{2}) - u(0)) + c \cdot \sigma(u(0) - u(\frac{1}{2})) + \frac{1-c}{2} \cdot \sigma(u(\frac{1}{2}) - u(1)) \leq \frac{1}{2}$$
$$\iff \frac{1-c}{2} \cdot \sigma(-(\frac{1}{2} + v)) + c \cdot \sigma(\frac{1}{2} + v) + \frac{1-c}{2} \cdot \sigma(\frac{1}{2} - v) \leq \frac{1}{2}$$
$$\iff \frac{1-c}{2}\left(1 - \sigma(\frac{1}{2} + v)\right) + c \cdot \sigma(\frac{1}{2} + v) + \frac{1-c}{2} \cdot \sigma(\frac{1}{2} - v) \leq \frac{1}{2}$$
$$\iff c\left(-\frac{1}{2}\left(1 - \sigma(\frac{1}{2} + v)\right) + \sigma(\frac{1}{2} + v) - \frac{1}{2}\sigma(\frac{1}{2} - v)\right)$$
$$\leq \frac{1}{2} - \frac{1}{2}\left(1 - \sigma(\frac{1}{2} + v)\right) - \frac{1}{2}\sigma(\frac{1}{2} - v)$$
$$\iff c\left(-\left(1 - \sigma(\frac{1}{2} + v)\right) + 2\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)\right) \leq \sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)$$
$$\iff c\left(3\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v) - 1\right) \leq \sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v). \tag{6}$$

If $v = 0$, then the right-hand side of inequality (6) is clearly equal to zero while the left-hand side is also equal to zero since $0 \leq c \leq \bar{c}(0, \sigma) = 0$, thus showing the statement

for $v = 0$. If $v > 0$, then since

$$3\sigma(\tfrac{1}{2} + v) - \sigma(\tfrac{1}{2} - v) - 1 > 2\sigma(\tfrac{1}{2} + v) - 1 > 2 \cdot \tfrac{1}{2} - 1 = 0,$$

the inequality (6) is equivalent to

$$c \le \frac{\sigma(\tfrac{1}{2} + v) - \sigma(\tfrac{1}{2} - v)}{3\sigma(\tfrac{1}{2} + v) - \sigma(\tfrac{1}{2} - v) - 1} = \left(2 + \frac{\sigma(\tfrac{1}{2} + v) + \sigma(\tfrac{1}{2} - v) - 1}{\sigma(\tfrac{1}{2} + v) - \sigma(\tfrac{1}{2} - v)}\right)^{-1} = \bar{c}(v, \sigma), \quad (7)$$

showing the statement for $v > 0$.

Part 2. We consider two cases. First, assume $v \ge 0$ and $c > \bar{c}(v, \sigma)$. Then, by inequality (7), we have $P(0, \tfrac{1}{2}) = 1 - P(\tfrac{1}{2}, 0) < \tfrac{1}{2}$. By symmetry, $P(1, \tfrac{1}{2}) < \tfrac{1}{2}$. Meanwhile, it is clear that $P(\tfrac{1}{2}, \tfrac{1}{2}) = \tfrac{1}{2}$. Hence, $(\tfrac{1}{2}, \tfrac{1}{2})$ is a Nash equilibrium of the game with $\epsilon = 0$. Moreover, it trivially satisfies "the additional property" in the statement of Proposition 4 since $P(x_i', \tfrac{1}{2}) < P(\tfrac{1}{2}, \tfrac{1}{2})$ for all $x_i' \ne \tfrac{1}{2}$. Therefore, by Proposition 4, we conclude that $(\tfrac{1}{2}, \tfrac{1}{2})$ is a Nash equilibrium for $\epsilon > 0$.

Next assume $v < 0$. From the calculation in Part 1, we have

$$P(\tfrac{1}{2}, 0) > \frac{1}{2}$$
$$\iff \frac{1 - c}{2}\left(1 - \sigma(\tfrac{1}{2} + v)\right) + c\sigma(\tfrac{1}{2} + v) + \frac{1 - c}{2}\sigma(\tfrac{1}{2} - v) > \frac{1}{2}$$
$$\iff \frac{1 - c}{2}\left(1 - \sigma(\tfrac{1}{2} + v) + \sigma(\tfrac{1}{2} - v)\right) + c\sigma(\tfrac{1}{2} + v) > \frac{1}{2}.$$

Since $\sigma(\tfrac{1}{2} + v) \ge \sigma(0) = \tfrac{1}{2}$ for any v, the last inequality holds if

$$1 - \sigma(\tfrac{1}{2} + v) + \sigma(\tfrac{1}{2} - v) > 1$$
$$\iff \sigma(\tfrac{1}{2} - v) > \sigma(\tfrac{1}{2} + v)$$
$$\iff v < 0.$$

Therefore, because $v < 0$ by assumption, $P(0, \tfrac{1}{2}) = 1 - P(\tfrac{1}{2}, 0) < \tfrac{1}{2}$ while $P(\tfrac{1}{2}, \tfrac{1}{2}) = \tfrac{1}{2}$ by symmetry. Again, by Proposition 4, $(\tfrac{1}{2}, \tfrac{1}{2})$ is a Nash equilibrium. $\qquad \square$

A.4 Proof of Proposition 5

Proof. We show the first part and the second part of the proposition in sequence.

Recall that

$$\bar{c}(v, \sigma) = \left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}\right)^{-1}.$$

Part 1. First, $\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)$ is increasing in v by the assumption that σ is increasing. Moreover, $\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v)$ is nonincreasing in v by weak concavity of σ in the positive domain. Thus $\bar{c}(v, \sigma)$ is strictly increasing in v.

As $v \to 0$, $\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v) \to 0$. Meanwhile $\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v)$ goes to $2\sigma(\frac{1}{2} + v)$, which is strictly larger than 1 as $\sigma(\frac{1}{2}) > \frac{1}{2}$. Therefore $\bar{c}(v, \sigma)$ goes to 0 as $v \to 0$.

Finally, $\lim_{v \nearrow \frac{1}{2}} \bar{c}(v, \sigma) = \bar{c}(\frac{1}{2}, \sigma) = \frac{1}{3}$.

Part 2. (a) To prove the claim, it suffices to show that

$$\frac{\sigma^\rho(\frac{1}{2} + v) + \sigma^\rho(\frac{1}{2} - v) - 1}{\sigma^\rho(\frac{1}{2} + v) - \sigma^\rho(\frac{1}{2} - v)} = \frac{\sigma(\rho[\frac{1}{2} + v]) + \sigma(\rho[\frac{1}{2} - v]) - 1}{\sigma(\rho[\frac{1}{2} + v]) - \sigma(\rho[\frac{1}{2} - v])},$$

is strictly increasing in $\rho > 0$. We shall show this holds. To do so, first differentiate the above expression with respect to ρ, to obtain

$$\frac{(\frac{1}{2} - v)\sigma'(\rho[\frac{1}{2} - v])(2\sigma(\rho[\frac{1}{2} + v]) - 1) - (\frac{1}{2} + v)\sigma'(\rho[\frac{1}{2} + v])(2\sigma(\rho[\frac{1}{2} - v]) - 1)}{\left(\sigma(\rho[\frac{1}{2} + v]) - \sigma(\rho[\frac{1}{2} - v])\right)^2}.$$

Since the denominator of this expression is always positive, it suffices to show that the numerator of this expression is positive. It is positive if and only if

$$\frac{\sigma(\rho[\frac{1}{2} + v]) - \frac{1}{2}}{\frac{1}{2} + v} \cdot \frac{1}{\sigma'(\rho[\frac{1}{2} + v])} > \frac{\sigma(\rho[\frac{1}{2} - v]) - \frac{1}{2}}{\frac{1}{2} - v} \cdot \frac{1}{\sigma'(\rho[\frac{1}{2} - v])},$$

which is equivalent to

$$\frac{\sigma(\rho[\frac{1}{2} + v]) - \frac{1}{2}}{\rho[\frac{1}{2} + v]} \cdot \frac{1}{\sigma'(\rho[\frac{1}{2} + v])} > \frac{\sigma(\rho[\frac{1}{2} - v]) - \frac{1}{2}}{\rho[\frac{1}{2} - v]} \cdot \frac{1}{\sigma'(\rho[\frac{1}{2} - v])}.$$

This inequality holds if

$$\frac{\sigma(x) - \frac{1}{2}}{x} \cdot \frac{1}{\sigma'(x)} = \frac{\sigma(x) - \frac{1}{2}}{x\sigma'(x)}$$

is strictly increasing in x for all $x \geq 0$. Noting that $\sigma(0) = \frac{1}{2}$, this follows from the increasing average-marginal ratio property, proving the claim.

(b) First we show that $\bar{c}(v, \sigma) \leq \frac{2v}{4v+1}$. It suffices to show that

$$\frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)} \geq \frac{1}{2v}. \tag{8}$$

To see this, note that the weak concavity of σ for the positive domain implies that

$$\frac{\sigma(\frac{1}{2} - v) - \frac{1}{2}}{\frac{1}{2} - v} \geq \frac{\sigma(\frac{1}{2} + v) - \frac{1}{2}}{\frac{1}{2} + v}.$$

This is equivalent to

$$(\frac{1}{2} + v)(\sigma(\frac{1}{2} - v) - \frac{1}{2}) \geq (\frac{1}{2} - v)(\sigma(\frac{1}{2} + v) - \frac{1}{2})$$
$$\iff v(\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1) \geq \frac{1}{2}(\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v))$$
$$\iff \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)} \geq \frac{1}{2v}, \tag{9}$$

proving the claim.

Now we show that $\lim_{\rho \searrow 0} \bar{c}(v, \sigma) = \frac{2v}{4v+1}$, when σ is differentiable. Since $\sigma^\rho(x) = \sigma(\rho x)$, we have that

$$\frac{\sigma^\rho(\frac{1}{2} + v) + \sigma^\rho(\frac{1}{2} - v) - 1}{\sigma^\rho(\frac{1}{2} + v) - \sigma^\rho(\frac{1}{2} - v)} = \frac{\sigma(\rho[\frac{1}{2} + v]) + \sigma(\rho[\frac{1}{2} - v]) - 1}{\sigma(\rho[\frac{1}{2} + v]) - \sigma(\rho[\frac{1}{2} - v])},$$

and both the numerator and the denominator of the right hand side goes to zero as $\rho \to 0$. By this fact and the differentiability of σ, we apply the L'Hopital's theorem (differentiating with respect to ρ) to obtain

$$\lim_{\rho \to 0} \frac{\sigma(\rho[\frac{1}{2} + v]) + \sigma(\rho[\frac{1}{2} - v]) - 1}{\sigma(\rho[\frac{1}{2} + v]) - \sigma(\rho[\frac{1}{2} - v])} = \lim_{\rho \to 0} \frac{(\frac{1}{2} + v)\sigma'(\rho[\frac{1}{2} + v]) + (\frac{1}{2} - v)\sigma'(\rho[\frac{1}{2} - v])}{(\frac{1}{2} + v)\sigma'(\rho[\frac{1}{2} + v]) - (\frac{1}{2} - v)\sigma'(\rho[\frac{1}{2} - v])}$$
$$= \frac{(\frac{1}{2} + v)\sigma'(0) + (\frac{1}{2} - v)\sigma'(0)}{(\frac{1}{2} + v)\sigma'(0) - (\frac{1}{2} - v)\sigma'(0)}$$
$$= \frac{1}{2v}.$$

Since $(2 + \frac{1}{2v})^{-1} = \frac{2v}{4v+1}$, we have the desired result.

(c) Take a sequence $\{\sigma_n\}_{n=1}^\infty$ that converges pointwise to σ^*. Notice that $\lim_{n \to \infty} \sigma_n(\frac{1}{2} +$

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$v) = \lim_{n \to \infty} \sigma_n(\frac{1}{2} - v) = 1$. This means that

$$\lim_{n \to \infty} \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)} = \infty,$$

so $\lim_{n \to \infty} \bar{c}(v, \sigma_n) = 0$.

$\square$

A.5 Proofs of Proposition 2 and Theorem 2

Before showing the results, first note that $W(0) = W(1)$ by symmetry, and

$$W(0) \geq W(\frac{1}{2})$$

$$\iff \frac{1-c}{2} \cdot u(0) + c \cdot u(\frac{1}{2}) + \frac{1-c}{2} \cdot u(1) \geq \frac{1-c}{2} \cdot u(\frac{1}{2}) + c \cdot u(0) + \frac{1-c}{2} \cdot u(\frac{1}{2})$$

$$\iff \frac{1-c}{2} \cdot 1 + c \cdot \left(\frac{1}{2} - v\right) + \frac{1-c}{2} \cdot 0 \geq \frac{1-c}{2} \cdot \left(\frac{1}{2} - v\right) + c \cdot 1 + \frac{1-c}{2} \cdot \left(\frac{1}{2} - v\right)$$

$$\iff 2(1-c) + 2c(1-2v) \geq (1-c)(1-2v) + 4c + (1-c)(1-2v)$$

$$\iff c(-2 + 2(1-2v) + (1-2v) - 4 + (1-2v)) \geq 2(1-2v) - 2$$

$$\iff c(4(1-2v) - 6) \geq -4v$$

$$\iff c(1 + 4v) \leq 2v. \tag{10}$$

Proof of Theorem 2. By Theorem 1, the assumption that $(0, 1)$ is a Nash equilibrium implies $v \geq 0$. When $v \geq 0$, inequality (10) is equivalent to $c \leq \frac{2v}{1+4v} := \hat{c}(v)$. Thus it suffices to show $\bar{c}(v, \sigma) \leq \hat{c}(v)$ for all $v \geq 0$ and σ. This inequality $\bar{c}(v, \sigma) \leq \hat{c}(v)$ holds for $v = 0$ since $\bar{c}(0, \sigma) = \hat{c}(0) = 0$. If $v > 0$, by algebraic manipulation we obtain

$$\bar{c}(v, \sigma) \leq \hat{c}(v)$$

$$\iff \left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}\right)^{-1} \leq \frac{2v}{1 + 4v}$$

$$\iff 1 + 4v \leq 2v\left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}\right)$$

$$\iff 1 \leq 2v \times \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}$$

$$\iff \frac{1}{2v} \leq \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)},$$

which holds by inequality (8). This completes the proof. $\square$

Proof of Proposition 2. If $v \geq 0$, then the statement of the Proposition is a special case of Theorem 2 since $c = 0 \leq \bar{c}(v, \sigma)$ for the perfectly polarized distribution. So assume $v < 0$. Recall that $c = 0$ holds for the perfectly polarized distribution. This implies that inequality (10) is violated, since its left hand side is zero and its right hand side is strictly negative. Therefore $W(\frac{1}{2}) > W(0) = W(1)$. By Proposition 1, the unique Nash equilibrium is $(\frac{1}{2}, \frac{1}{2})$, thus completing the proof. $\square$

A.6 Proof of Theorem 3

Proof. Let (x_A^*, x_B^*) be the strategy profile as defined in the statement of the Theorem. For any k such that u_k is convex, by definition we have

$$u_k(0) + u_k(1) \geq 2u_k(\frac{1}{2}).$$

For any other k, u_k is strictly concave and we have

$$u_k(0) + u_k(1) < 2u_k(\frac{1}{2}).$$

Therefore, for any $i \in \{A, B\}$ and $j \neq i, x, y \in \{0, 1\}^n$ with $y_k = 1 - x_k$ for all k and any $x_i' = (x_{i1}', \ldots, x_{in}') \in \mathcal{P}$,

$$\sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - x_k|) + \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - y_k|) \leq \sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - x_k|) + \sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - y_k|),$$

with strict inequality if there exists k such that $x_{ik}' \neq \frac{1}{2}$ and u_k is strictly concave. Since σ is strictly increasing, this inequality implies

$$\sigma\left(-\left[\sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - x_k|)\right]\right)$$

$$\leq \sigma\left(\sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - y_k|)\right).$$

Thus by the symmetry assumption of σ (that is, $\sigma(t) + \sigma(-t) = 1$ for all $t \in \mathbb{R}$), we have

$$\sigma\left(\sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - x_k|)\right)$$

$$+\sigma\left(\sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - y_k|)\right) \geq 1,$$

with strict inequality if there exists k such that $x_{ik}' \neq \frac{1}{2}$ and u_k is strictly concave.

Hence by the symmetry of f, we have

$$\sum_{x \in \mathcal{P}} \sigma\left(\sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - x_k|)\right) f(x) \geq \frac{1}{2}$$

with strict inequality if there exists k such that $x_{ik}' \neq \frac{1}{2}$ and u_k is strictly concave. This means that $P(x_j^*, x_i') \geq \frac{1}{2}$, and symmetry implies that $P(x_i', x_j^*) \leq \frac{1}{2}$. Therefore, the only possible deviation x_i' by i from x_i^* that does not reduce her winning probability is one where $x_{ik}' \neq \bar{x}_{ik}$ for some k's such that u_k's are convex, while $x_{ik}' = x_{ik}^*$ for all other k's. But this deviation would keep the winning probability for i unchanged at $\frac{1}{2}$ at best while strictly reducing her payoff when she wins, thus it is not a profitable deviation.

Uniqueness of the Nash equilibrium holds by an analogous argument as in the proof of Proposition 1 and Theorem 1 and hence is omitted. $\square$

A.7 Increasing Average-Marginal Ratio Properties in Logit and Probit Models

In this section we prove the claim made in the main text when we implement the comparative statics in Proposition 5, which is summarized in the following proposition:

Proposition 7. *1. The logistic voting function $\hat{\sigma}^\lambda$ satisfies the increasing average-marginal ratio property for any λ.*

2. The normal voting function $\tilde{\sigma}^p$ satisfies the increasing average-marginal ratio property for any p.

Proof. **Part 1.** Let $g(t) = \frac{\tau(t)}{\tau'(t)t}$, where $\tau(t) = \hat{\sigma}^\lambda(t) - \hat{\sigma}^\lambda(0)$. By computation,

$$g(t) = \frac{e^{\lambda t} - e^{-\lambda t}}{2\lambda t}.$$

The derivative $g'(t)$ of function $g(t)$ is

$$g'(t) = \frac{2\lambda t[\lambda e^{\lambda t} + \lambda e^{-\lambda t}] - 2\lambda[e^{\lambda t} - e^{-\lambda t}]}{(2\lambda t)^2}.$$

Since the denominator of $g'(t)$ is positive for all $t > 0$ and 2λ is constant, it suffices to show that

$$h(t) := t[\lambda e^{\lambda t} + \lambda e^{-\lambda t}] - [e^{\lambda t} - e^{-\lambda t}]$$

is positive for all $t > 0$. To show this, first note that $h(0) = 0$. Moreover,

$$\begin{aligned} h'(t) &= [\lambda e^{\lambda t} + \lambda e^{-\lambda t}] + t\lambda^2[e^{\lambda t} - e^{-\lambda t}] - [\lambda e^{\lambda t} + \lambda e^{-\lambda t}] \\ &= t\lambda^2[e^{\lambda t} - e^{-\lambda t}]. \end{aligned}$$

This expression is obviously nonnegative for all $t \geq 0$ and strictly positive for all $t > 0$. Thus $h(t)$ is positive for all $t > 0$, thus so is $g'(t)$, which implies that $g(t)$ is increasing for $t \geq 0$.

Part 2. Let $g(t) = \frac{\tau(t)}{\tau'(t)t}$, where $\tau(t) = \tilde{\sigma}^p(t) - \tilde{\sigma}^p(0)$ and $\tilde{\sigma}^p$ is the cumulative distribution function of a normal distribution with mean zero and standard deviation $s := 1/p$, $N(0, s^2)$. The derivative of g is

$$g'(t) = \frac{\tau'(t)\tau'(t)t - \tau(t)[\tau''(t)t + \tau'(t)]}{(\tau'(t)t)^2}.$$

Because the denominator of $g'(t)$ is positive for all $t > 0$, we focus on the numerator

$$h(t) = \tau'(t)\tau'(t)t - \tau(t)[\tau''(t)t + \tau'(t)],$$

and shall show that $h(t)$ is positive for all $t > 0$. Since

$$\tau'(t) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{t^2}{2s^2}}$$

by definition, we have

$$\tau''(t) = -\frac{t}{s^2}\tau'(t).$$

Substituting this into the definition of $h(t)$, we obtain

$$h(t) = \tau'(t)\tau'(t)t - \tau(t)[-\frac{1}{s^2}\tau'(t)t^2 + \tau'(t)]$$
$$= \tau'(t)[\tau'(t)t + \frac{1}{s^2}\tau(t)t^2 - \tau(t)].$$

We shall show that $h(t)$ is positive for all $t > 0$. To see this, first note that $\tau'(t)$ is positive for all $t > 0$. So it suffices to show that

$$i(t) = \tau'(t)t + \frac{1}{s^2}\tau(t)t^2 - \tau(t)$$

is positive for all $t > 0$. To show this, first note that $i(0) = 0$. Second, differentiating $i(\cdot)$ we obtain

$$i'(t) = \tau''(t)t + \tau'(t) + \frac{1}{s^2}\tau'(t)t^2 + \frac{2}{s^2}\tau(t)t - \tau'(t)$$
$$= -\frac{1}{s^2}\tau'(t)t^2 + \frac{1}{s^2}\tau'(t)t^2 + \frac{2}{s^2}\tau(t)t$$
$$= \frac{2}{s^2}\tau(t)t$$
$$> 0.$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

A.8 Generalization to the case where σ is nondecreasing

The main text assumed that the voting function σ is strictly increasing. As mentioned in footnote 13 in the main text, our results can be generalized for any nondecreasing voting function σ at the cost of more complicated statements of the results. To illustrate this point, in this section we present a generalization of Theorem 1 under the assumption that σ is nondecreasing. Similar extensions can be made for other results, although we omit them for brevity.

First, define
$$\bar{c}(v, \sigma) = \left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}\right)^{-1}$$
for any v such that $\sigma(\frac{1}{2} + v) > \sigma(\frac{1}{2} - v)$, and $\bar{c}(v, \sigma) = 0$ otherwise.

Theorem 1'. *1. If $\sigma(\frac{1}{2} - v) \leq \sigma(\frac{1}{2} + v)$ (which holds if voters' utility function is*

convex) and $c \leq \bar{c}(v, \sigma)$ then $(0, 1)$ is a unique Nash equilibrium.

2. Otherwise, $(\frac{1}{2}, \frac{1}{2})$ is a unique Nash equilibrium.

Proof of Theorem 1'. As in the proof of Theorem 1, if there exists a Nash equilibrium, then it is unique. Note that the argument did not rely on strict increasingness of σ.

Part 1. By Proposition 4, the assumption $(\bar{x}_A, \bar{x}_B) = (0, 1)$, and symmetry, $(0, 1)$ is a Nash equilibrium if and only if the vote share of candidate B is at most $\frac{1}{2}$ when she chooses position $\frac{1}{2}$ while candidate A chooses 0. By the argument in the proof of Theorem 1, this condition is equivalent to:

$$P(\frac{1}{2}, 0) \leq \frac{1}{2}$$
$$\iff c \left(3\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v) - 1 \right) \leq \sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v). \tag{11}$$

If $\sigma(\frac{1}{2} + v) = \sigma(\frac{1}{2} - v)$, then the right-hand side of inequality (11) is clearly equal to zero while the left-hand side is also equal to zero since $0 \leq c \leq \bar{c}(v, \sigma) = 0$, thus showing the statement for any v such that $\sigma(\frac{1}{2} + v) = \sigma(\frac{1}{2} - v)$. If $\sigma(\frac{1}{2} - v) < \sigma(\frac{1}{2} + v)$, then since

$$3\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v) - 1 > 2\sigma(\frac{1}{2} + v) - 1 \geq 2 \cdot \frac{1}{2} - 1 = 0,$$

the inequality (6) is equivalent to

$$c \leq \frac{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)}{3\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v) - 1} = \left(2 + \frac{\sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) - 1}{\sigma(\frac{1}{2} + v) - \sigma(\frac{1}{2} - v)} \right)^{-1} = \bar{c}(v, \sigma), \tag{12}$$

showing the statement for any v such that $\sigma(\frac{1}{2} - v) < \sigma(\frac{1}{2} + v)$.

Part 2. We consider two cases. First, assume $\sigma(\frac{1}{2} - v) \leq \sigma(\frac{1}{2} + v)$ and $c > \bar{c}(v, \sigma)$. Then, by inequality (12), we have $P(0, \frac{1}{2}) = 1 - P(\frac{1}{2}, 0) < \frac{1}{2}$. By symmetry, $P(1, \frac{1}{2}) < \frac{1}{2}$. Meanwhile, it is clear that $P(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$. Hence, $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium of the game with $\epsilon = 0$. Moreover, it trivially satisfies "the additional property" in the statement of Proposition 4 since $P(x'_i, \frac{1}{2}) < P(\frac{1}{2}, \frac{1}{2})$ for all $x'_i \neq \frac{1}{2}$. Therefore, by Proposition 4, we conclude that $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium for $\epsilon > 0$.

Next assume $\sigma(\frac{1}{2} - v) > \sigma(\frac{1}{2} + v)$. From the calculation in Part 1, we have

$$P(\frac{1}{2}, 0) > \frac{1}{2}$$
$$\iff \frac{1-c}{2}\left(1 - \sigma(\frac{1}{2} + v)\right) + c\sigma(\frac{1}{2} + v) + \frac{1-c}{2}\sigma(\frac{1}{2} - v) > \frac{1}{2}$$
$$\iff \frac{1-c}{2}\left(1 - \sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v)\right) + c\sigma(\frac{1}{2} + v) > \frac{1}{2}.$$

Since $\sigma(\frac{1}{2} + v) \geq \sigma(0) = \frac{1}{2}$ for any v, the last inequality holds if

$$1 - \sigma(\frac{1}{2} + v) + \sigma(\frac{1}{2} - v) > 1$$
$$\iff \sigma(\frac{1}{2} - v) > \sigma(\frac{1}{2} + v).$$

Therefore, because $\sigma(\frac{1}{2} - v) > \sigma(\frac{1}{2} + v)$ by assumption, $P(0, \frac{1}{2}) = 1 - P(\frac{1}{2}, 0) < \frac{1}{2}$ while $P(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$ by symmetry. Again, by Proposition 4, $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium. $\square$

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