Online Supplementary Appendix to: "Optimal Timing of Policy Announcements in Dynamic Election Campaigns"

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I Definition of Bayes Rule

Fix candidate i's history $h_i^t = \left(\left(t_i^k, X_i^k \right)_{k=1}^{k_i}, \left(t_j^l, X_j^l \right)_{l=1}^{l_j}, t, z_i \right)$ arbitrarily. If $t = T$, then candidate i believes that $h_j^t = (\emptyset, \emptyset, T, no)$ with probability one. Hence, we focus on $t < T$. Define $\left(t_i^l, X_i^l \right)_{l=1}^{l_i}$ as follows: Let t_i^1 be the smallest time $t \in \{ t_i^1, ..., t_i^{k_i} \}$ such that, for k with $t = t_i^k$, $X_i^k \neq X_i^0$ holds (that is, $-t_i^1$ is the first time for candidate i to change her policy set); given t_i^1, let t_i^2 be the smallest time $t \in \{ t_i^1, ..., t_i^{k_i} \}$ such that $t > t_i^1$ and for k with $t = t_i^k$, $X_i^k \neq X_i^{k-1}$ holds (that is, $-t_i^2$ is the second time for candidate i to change her policy set), and so on.

When those conditions are met, we say that $\left(t_i^l, X_i^l \right)_{l=1}^{l_i}$ is **compatible** with $\left(t_i^k, X_i^k \right)_{k=1}^{k_i}$.

Given h_i^t, let $\vec{\theta}(h_i^t) := \left(\left(t_i^l, X_i^l \right)_{l=1}^{l_i}, \left(t_j^l, X_j^l \right)_{l=1}^{l_j} \right)$.

Fix $\left(t_i^l, X_i^l \right)_{l=1}^{l_i}$. Suppose that there exists $\left(t_j^k, X_j^k \right)_{k=1}^{k_j}$ with which $\left(t_j^l, X_j^l \right)_{l=1}^{l_j}$ is compatible, such that

$$ h_j^t = \left(\left(t_i^l, X_i^l \right)_{l=1}^{l_i}, \left(t_j^k, X_j^k \right)_{k=1}^{k_j}, t, no \right) $$

happens with a positive probability by σ_j^* conditional on the realization of $\left(t_j^k \right)_{k=1}^{k_j}$, $\left(t_i^l, X_i^l \right)_{l=1}^{l_i}$, and t:[1] At each time t_j^k for $k = 1, ..., k_j$, given candidate j's history $h_j^{t_j^k} = \left(\left(t_i^l, X_i^l \right)_{l=1}^{l(t_j^k)}, \left(t_j^{k'}, X_j^{k'} \right)_{k'=1}^{k-1}, t_j^k, yes \right)$ with $l\left(t_j^k \right)$ being the largest l with $t_i^l < t_j^k$ (that is, $\left(t_i^l, X_i^l \right)_{l=1}^{l(t_j^k)}$ is compatible with $\left(t_i^l, X_i^l \right)_{l=1}^{l_i}$), $\sigma_j^*(h_j^{t_j^k})\left(X_j^k \right) > 0$. Let $H_j^{\sigma_j^*}\left(h_i^t \right)$ be the set of candidate j's history satisfying this condition.

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[1]Given candidate i's history, she believes that candidate j receives an opportunity at $-t$ with probability 0 when candidate i receives an opportunity. Hence, we require that the last element is *no*.

Suppose first that $H_j^{\sigma_j^*}\left(h_i^t\right) \neq \emptyset$. Given a history h_i^t (note that this determines $\left(t_i^l, X_i^l\right)_{l=1}^{l_i}$ and $\left(t_j^l, X_j^l\right)_{l=1}^{l_j}$) and a set $\hat{H}_j\left(h_i^t\right) \subseteq H_j^{\sigma_j^*}\left(h_i^t\right)$, we can classify $h_j^t \in \hat{H}_j\left(h_i^t\right)$ into the following subsets: Given h_i^t and $\hat{H}_j\left(h_i^t\right)$, let KX_j be the set of k_j and $\left(X_j^k\right)_{k=1}^{k_j}$ such that there exists $(t_j^k)_{k=1}^{k_j}$ such that $\left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right) \in \hat{H}_j\left(h_i^t\right)$. Given h_i^t, $\hat{H}_j\left(h_i^t\right)$, and $(k_j, \left(X_j^k\right)_{k=1}^{k_j}) \in KX_j$, let T_j^1, $T_j^2\left(t_j^1\right)$, ..., $T_j^{k_j}(t_j^1, ..., t_j^{k_j-1})$ be, respectively, the set of t_j^1 such that there exists $(t_j^2, ..., t_j^{k_j})$ such that $\left(t_j^l, X_j^l\right)_{l=1}^{l_j}$ is compatible with $\left(t_j^k, X_j^k\right)_{k=1}^{k_j}$ and $\left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right) \in \hat{H}_j\left(h_i^t\right)$; the set of t_j^2 such that, given t_j^1, there exists $(t_j^3, ..., t_j^{k_j})$ such that $\left(t_j^l, X_j^l\right)_{l=1}^{l_j}$ is compatible with $\left(t_j^k, X_j^k\right)_{k=1}^{k_j}$ and $\left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right) \in \hat{H}_j\left(h_i^t\right)$; and so on, up to the set of $t_j^{k_j}$ such that, given $t_j^1, t_j^2, \ldots, t_j^{k-1}$, $\left(t_j^l, X_j^l\right)_{l=1}^{l_j}$ is compatible with $\left(t_j^k, X_j^k\right)_{k=1}^{k_j}$ and $\left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right) \in \hat{H}_j\left(h_i^t\right)$.[2] Given h_i^t and $H_j^{\sigma_j^*}\left(h_i^t\right)$, define KX_j^*, $T_j^{1,*}$, $T_j^{2,*}\left(t_j^1\right)$, ..., $T_j^{k_j,*}(t_j^1, ..., t_j^{k_j-1})$ in a similar manner, where we replace $\hat{H}_j\left(h_i^t\right)$ with $H_j^{\sigma_j^*}\left(h_i^t\right)$. Define $\bar{g} = g(h_j^t)$ where h_j^t has no opportunity for j and hence j's preparation is X, and (t_i^l, X_i^l) is equal to the part of $(t_i^k, X_i^k)_{k=1}^{k_i}$ in h_i^t where there is a change of i's policy set. Then, given h_i^t and $\hat{H}_j\left(h_i^t\right)$, for any $g\left(h_j^t\right)$, we define

$$\int_{h_j^t \in \hat{H}_j\left(h_i^t\right)} g\left(h_j^t\right) d\beta_i\left(h_j^t | h_i^t\right) := \frac{A}{B}, \tag{18}$$

where

$$A = e^{-\lambda t} \cdot \mathbb{I}_{(0,\cdot) \in KX_j} \bar{g}$$

$$+ \sum_{\left(k_j, (X_j^k)_{k=1}^{k_j}\right) \in KX_j} \int_{t_j^1 \in T_j^1} \int_{t_j^2 \in T_j^2(t_j^1)} \cdots \int_{t_j^{k_j} \in T_j^{k_j}(t_j^1, ..., t_j^{k_j-1})} \left(\begin{array}{c} g\left(h_j^t\right) e^{-(T-t)\lambda} \frac{((T-t)\lambda)^{k_j}}{k_j!} \\ \times \prod_{k=1}^{k_j} \sigma_j^*(h_j^{t_j^k})\left(X_j^k\right) \end{array} \right) dt_j^{k_j} \cdots dt_j^1,$$

and

$$B = e^{-\lambda t} \cdot \mathbb{I}_{(0,\cdot) \in KX_j^*}$$

$$+ \sum_{\left(k_j, (X_j^k)_{k=1}^{k_j}\right) \in KX_j^*} \int_{t_j^1 \in T_j^{1,*}} \int_{t_j^2 \in T_j^{2,*}(t_j^1)} \cdots \int_{t_j^{k_j} \in T_j^{k_j,*}(t_j^1, ..., t_j^{k_j-1})} \left(\begin{array}{c} e^{-(T-t)\lambda} \frac{((T-t)\lambda)^{k_j}}{k_j!} \\ \times \prod_{k=1}^{k_j} \sigma_j^*(h_j^{t_j^k})\left(X_j^k\right) \end{array} \right) dt_j^{k_j} \cdots dt_j^1.$$

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[2]For simpler notation, we suppress the dependence of KX_j on h_i^t and $\hat{H}_j\left(h_i^t\right)$ and the dependence of T_j^1, $T_j^2\left(t_j^1\right)$, ..., $T_j^{k_j}(t_j^1, ..., t_j^{k_j-1})$ on h_i^t, $\hat{H}_j\left(h_i^t\right)$, and $(k_j, \left(X_j^k\right)_{k=1}^{k_j})$.

For example, given a fixed continuation strategy profile σ and h_i^t, the function $g\left(h_j^t\right)$ can be candidate i's continuation payoff $u_i\left(\sigma|h_i^t, h_j^t\right)$.

If $H_j^{\sigma_j^*}\left(h_i^t\right) = \emptyset$ (h_i^t cannot be explained without j's deviation), then $d\beta_i\left(h_j^t|h_i^t\right)$ is arbitrary, as long as the following three conditions are satisfied. To state the first condition, let $H_j(h_i^t)$ be the set of candidate j's histories of the form

$$h_j^t = \left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right)$$

such that $\left(t_j^l, X_j^l\right)_{l=1}^{l_j}$ is compatible with $\left(t_j^k, X_j^k\right)_{k=1}^{k_j}$. The first condition is that $\int_{h_j^t \in H_j(h_i^t)} d\beta_i\left(h_j^t|h_i^t\right) = 1$.

The remaining two conditions constitute an analogue of the "no signaling what you don't know" condition.[3]

Specifically, the second condition is that, for each pair of histories for candidate i, $h_i^t, \tilde{h}_i^t \in H_i$ with $\vec{\theta}(h_i^t) = \vec{\theta}(\tilde{h}_i^t) := \vec{\theta}$, we have that for each $h_j^t \in H_j$,

$$\beta_i\left(h_j^t|h_i^t\right) = \beta_i\left(h_j^t|\tilde{h}_i^t\right) := \beta_i\left(h_j^t|\vec{\theta}\right). \tag{19}$$

Note that this condition automatically holds if $H_j^{\sigma_j^*}\left(h_i^t\right) \neq \emptyset$. Thus, what we are additionally requiring here is condition (19) for off the path of equilibrium play. Intuitively, candidate i believes that candidate j's deviation is not correlated with h_i^t beyond what is observable to candidate j, i.e., beyond $\vec{\theta}$.

The third condition is that, for each i, t, $(t_i^l, X_i^l)_{l=1}^{l_i}$, $X_i^{l_i+1}$, and $\left(t_j^k, X_j^k\right)_{k=1}^{k_j}$, for three histories

$$\begin{aligned} h_i^t &= \left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, no\right), \\ \hat{h}_i^t &= \left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t, yes\right), \text{ and} \\ \tilde{h}_i^{t'} &= \left(\left(t_i^l, X_i^l\right)_{l=1}^{l_i+1}, \left(t_j^k, X_j^k\right)_{k=1}^{k_j}, t', no\right) \text{ with } t_i^{l_i+1} = t, \end{aligned}$$

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[3] An analogue of Kreps and Wilson's (1982) structural consistency, Fudenberg and Tirole's (1991) reasonableness, and Watson's (2017) "plain PBE" would imply these two conditions.

we require that for each $h_j^t \in H_j$,

$$\beta_i \left(h_j^t | h_i^t \right) = \beta_i \left(h_j^t | \hat{h}_i^t \right) = \lim_{t' \searrow t} \beta_i \left(h_j^t | \tilde{h}_i^{t'} \right). \tag{20}$$

That is, candidate i's belief at a history at time $-t$ does not change depending on the arrival of the Poisson process or her own action X_i at time $-t$. Again, this condition automatically holds if $H_j^{\sigma_j^*} \left(h_i^t \right) \neq \emptyset$, and so the additional requirement here is about off the path of equilibrium play.

J   Proofs Omitted in Appendix C

J.1   Proof of Proposition 6

Fix any $t \in (-\infty, 0]$, and suppose that no candidate enters at any $-\tau \in (-t, 0]$. On the one hand, if candidate i enters at $-t$, her payoff is $v_{i,t}(\text{enter})$. By Assumption 2, $v_{i,t}(\text{enter}) \leq v_{i,0}(\text{enter})$. Since $v_{i,0}(\text{enter}) = v_i (x_i^*, X)$ by definition and $v_i (x_i^*, X) < v_i (X, X)$ as we are in Case 1, we have $v_{i,t}(\text{enter}) < v_i (X, X)$. On the other hand, if she does not enter, then her payoff is $v_i (X, X)$. Hence, it is uniquely optimal not to enter at $-t$. Since the payoffs are continuous in time due to the continuity of probabilities in time and the boundedness of v_i for each $i = A, B$, there exists $\varepsilon > 0$ such that no candidate enters for any time in $(-t - \varepsilon, -t]$. Hence the continuous-time backward induction implies the desired result.

J.2   The Formal Definition of $\bar{v}_{i,t}(\text{not})$

Fix t_0 that is defined in Remark 13. For $t > t_0$, define $\bar{v}_{i,t}(\text{not})$ as candidate i's expected continuation payoff at time $-t$ when she does not enter, assuming that each candidate will enter at times in $(-t, -t_0)$ upon receiving an opportunity. Such a payoff is well defined due to Assumption 1. It is formally defined by the following:

$$
\begin{aligned}
\bar{v}_{i,t}(\text{not}) \;=\; & e^{-(\lambda_i+\lambda_j)(t-t_0)} v_{i,t_0}(X, X) + e^{-\lambda_i(t-t_0)}(1 - e^{-\lambda_j(t-t_0)}) v_i(X, x_j^*) + (1 - e^{-\lambda_i(t-t_0)}) e^{-\lambda_j(t-t_0)} v_i(x_i^*, X) \\
& + \left(1 - e^{-\lambda_i(t-t_0)} \right) \left(1 - e^{-\lambda_j(t-t_0)} \right) \left(\frac{\lambda_i}{\lambda_i + \lambda_j} v_i^{BR_j} + \frac{\lambda_j}{\lambda_i + \lambda_j} \max_{X_i \in \mathcal{X}_i} v_i(X_i, x_j^*) \right).
\end{aligned}
$$

Let

$$t_i^* \equiv \inf \left\{ t > t_0 : \bar{v}_{i,t}(\text{not}) \geq v_{i,t}(\text{enter}) \right\}.$$

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Assumption 3 can be stated in the same way as in the main text.

J.3   Proof of Proposition 7

By the definition of t_0, there exists $\varepsilon > 0$ such that for all time in $(-t_0 - \varepsilon, -t_0]$, each candidate i enters under any PBE. Hence, if $t_i^* = t_j^* = \infty$, each candidate enters at all times in $(-\infty, -t_0]$. For the rest of the proof, we focus on the case in which at least one of t_A^* and t_B^* is less than ∞. Without loss, we assume $t_A^* \leq t_B^*$.

The following lemma shows that, for any PBE, candidate A does not enter at any time $-t < -t_A^*$:

Lemma 6. *Fix any σ_B such that (i) $\sigma_B(h_B^t) = x_B^*$ for any h_B^t with $\theta_B(h_B^t) = X$ for each $-t \in (-t_A^*, -t_0]$ and (ii) $\sigma_B(h_t) = BR_B(x_A)$ for any h_B^t with $\theta_A(h_B^t) = x_A$ for each $-t \in [-T, 0]$.[4] If σ_A is a best response to σ_B, then for any $h_t \in (X, X)$ with $-t < -t_A^*$, we have $\sigma_A(h_t)(X) = 1$.[5]*

The proof of the lemma is complicated, so we first assume that the lemma holds and show the proposition, and then prove the lemma. If $t_A^* = t_B^*$, then Lemma 6 implies Proposition 7 with $t_i = t_i^*$ for each i. Hence, we assume $t_A^* < t_B^*$.

Fix a PBE and, for each $i = A, B$, let $v_{i,t}(\text{not})$ be candidate i's continuation payoff at time $-t$ when i does not enter. Given Lemma 6, for $t \in [t_A^*, t_B^{**}]$ with t_B^{**} defined below, we calculate $v_{i,t}(\text{not})$ assuming that only candidate B enters in the time interval $(-t, -t_A^*)$ and both candidates enter in the time interval $[-t_A^*, -t_0]$. For $\tau \geq t$, Lemma 6 implies that candidate A does not enter at times in $(-\tau, -t)$. Hence, we have $v_{B,\tau}(\text{not}) \geq v_{B,t}(\text{not})$ for $\tau \geq t$ because candidate B at $-\tau$ can receive $v_{B,t}(\text{not})$ by committing to a strategy in which he keeps skipping opportunities from $-\tau$ to $-t$. Let

$$t_B^{**} \equiv \inf \left\{ t > t_0 : v_{B,t}(\text{not}) \geq v_{B,t}(\text{enter}) \right\}.$$

There are the following two cases: $t_B^{**} < \infty$ or $t_B^{**} = \infty$. The following lemma is useful:

Lemma 7. *If $t_B^{**} < \infty$, then $v_B(x_B^*, X) > v_B^{BR_A}$.*

Proof. Suppose otherwise. Then, Assumption 2 implies $v_B(x_B^*, X) = v_B^{BR_A}$. Then, $v_{B,t}(\text{enter})$ is constant in $t \in [t_0, \infty)$. At time $-t_B^{**} < -t_A^*$, there are the following three cases:

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[4]Recall the definition of $\theta_i(\cdot)$ from Section 4.3.

[5]Recall that "(X, X)" denotes the set of histories at which no candidates have entered.

1. Candidate A has the next opportunity at time $-t \in (-t_B^{**}, -t_A^*]$. Conditional on this event, candidate B obtains a payoff of $v_B^{BR_A} = v_B(x_B^*, X) = v_{B,t}(\text{enter})$ when he enters at $-t$ and a payoff of $v_{B,t}(\text{not})$ when he does not. Since t_B^{**} is the infimum of $t > t_0$ such that $v_{B,t}(\text{not}) \geq v_{B,t}(\text{enter})$, candidate B prefers to enter in this event.

2. Candidate B has the next opportunity at time $-t \in (-t_B^{**}, -t_A^*]$. Conditional on this event, since candidate B receives $v_{B,t}(\text{enter})$ at any $-t$ upon entering, candidate B is indifferent between entering and not entering.

3. No candidate has an opportunity at any time $-\bar{t} \in (-t_B^{**}, -t_A^*]$. Conditional on this event, candidate B strictly prefers to enter since $v_{B,t_A^*}(\text{enter}) > v_{B,t_A^*}(X, X)$.

Hence, it is uniquely optimal to enter at $-t_B^{**}$, which is a contradiction. □

Given this lemma, consider the following two cases:

1. $t_B^{**} < \infty$: In this case, we are left to prove Lemma 6. To see why, once we have shown Lemma 6, then for $t > t_B^{**}$, $v_{B,t}(\text{not}) \geq v_{B,t_B^{**}}(\text{not})$ holds since candidate B can skip opportunities until $-t_B^{**}$ without the opponent entering. Together with the fact that $v_{B,t}(\text{enter})$ is strictly decreasing in t by Lemma 7, we can conclude that candidate B does not enter at times in $(-\infty, -t_B^{**})$ in any PBE.

2. $t_B^{**} = \infty$: This means that candidate B enters at times in $(-\infty, 0]$ in any PBE given Lemma 6.

The argument so far proves that the proposition holds given Lemma 6. We now prove Lemma 6:

Proof of Lemma 6. Fix $\bar{\sigma}_A$, which is candidate A's strategy such that (i) if the current policy set profile is (X, X), then candidate A takes X for each $-t \in [-\bar{t}, -t_A^*)$ and takes $x_A^* \in X_A^*$ for each $-t \in (-t_A^*, 0]$, and (ii) if the current policy choice is (X, x_B) for some $x_B \in X$, then A takes a static best response against x_B.

Fix candidate A's history $h_A^{\bar{t}} \in (X, X)$ such that $-\bar{t} < -t_A^*$ and $z_{A,\bar{t}} = yes$.

We compare candidate A's payoff from entering at $x_A \in X_A^*$ and her payoff from taking $\bar{\sigma}_A$ under history $h_A^{\bar{t}}$. From candidate A's perspective at the history $h_A^{\bar{t}}$, there are two possibilities when candidate B takes σ_B that is fixed in the statement of Lemma 6, as follows:

1. Consider the realization of the Poisson process such that candidate B would not enter in $(-\bar{t}, -t_A^*]$ if $(\bar{\sigma}_A, \sigma_B)$ were taken.

   On the one hand, if A takes $\bar{\sigma}_A$, she obtains the value of $v_{A,t_A^*}(\text{not})$ by taking $\bar{\sigma}_A$.

   On the other hand, candidate A's payoff of entering at time $-\bar{t}$ is at most

   $$\max\left\{v_{A,t_A^*}(\text{enter}), v_A^{BR_B}\right\} = v_{A,t_A^*}(\text{enter}).$$

   The payoff $v_A^{BR_B}$ corresponds to the case where candidate B has an opportunity in $(-\bar{t}, -t_A^*]$.[6] Since $v_{A,t_A^*}(\text{not}) = v_{A,t_A^*}(\text{enter})$, it is weakly better for candidate A to take $\bar{\sigma}_A$ than entering at time $-\bar{t}$.

2. Consider the realization of the Poisson process such that candidate B would enter in $(-\bar{t}, -t_A^*]$ if $(\bar{\sigma}_A, \sigma_B)$ were taken. Let $\check{t}$ be the time at which candidate B would enter.

   On the one hand, A's continuation payoff from taking $\bar{\sigma}_A$ is $\hat{v}_{A,\check{t}}$, where

   $$\hat{v}_{A,t} := (1 - e^{-\lambda_A t})\left(\max_{X_A \in \mathcal{X}} v_A(X_A, x_B^*)\right) + e^{-\lambda_A t} v_A(X, x_B^*).$$

   On the other hand, her continuation payoff from entering at $-\bar{t}$ is $v_A^{BR_B}$.

   To compare these two values, it is instructive to examine why candidate A at $-t_A^*$ is indifferent between entering and not entering at histories in (X, X). Suppose now that candidate B has not entered at $-t_A^*$. There are following three events that can happen with positive probability until the deadline:

   (a) Candidate A receives the next opportunity at $-\tau > -t_A^*$: In this case, candidate A receives $v_{A,\tau}(\text{enter})$ at $-\tau$ regardless of candidate A's choice at $-t_A^*$. Note that, even if candidate A has entered before $-\tau$, since we assume that candidate A enters at some

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[6]Note that we only assume that candidate B would not enter in $(-\bar{t}, -t_A^*]$ if candidates took $(\bar{\sigma}_A, \sigma_B)$ but we do not assume that B would not enter given that A entered at $-\bar{t}$.

policy in X_A^*, the situation is that candidate A enters at some policy in X_A^* and candidate B has not at $-\tau$ (note that all the policies in X_A^* give rise to the same payoff).

(b) Candidate B receives the next opportunity at $-\tau > -t_A^*$: Candidate A receives a payoff of $v_A^{BR_B}$ (candidate B best-responds to x_A^* at $-\tau$) if she enters before $-t_A^*$; and $\hat{v}_{A,\tau}$ (candidate B enters while candidate A has not at $-\tau$) if she does not enter before $-t_A^*$.

(c) No candidate receives any opportunity in the time interval $(-t_A^*, -t_0]$: Candidate A receives $v_{A,t_0}(\text{enter})$ if she enters at $-t_1^*$; and $v_{A,t_0}(\text{not})$ if she does not at $-t_A^*$. We have assumed that $v_{A,t_0}(\text{enter}) > v_{A,t_0}(\text{not})$.

Note that candidate A is indifferent between entering and not entering at $-t_A^*$ in case (2a) and strictly prefers entering in case (2c). Since case (2c) happens with positive probability, it must be the case that candidate A strictly prefers not entering to entering in case (2b), in order for her to be indifferent between entering and not entering at $-t_A^*$. This implies that there exists $-\tilde{t} \in (-t_A^*, -t_0]$ such that $v_A^{BR_B} < \hat{v}_{A,\tilde{t}}$. Now, note that $\hat{v}_{A,t}$ is nondecreasing in t and $-\check{t} \leq -t_A^* < -\tilde{t}$ must hold. Hence, we have $\hat{v}_{A,\tilde{t}} \leq \hat{v}_{A,\check{t}}$, implying $v_A^{BR_B} < \hat{v}_{A,\check{t}}$.

Now we prove that candidate A does not enter at any history in (X, X) at any time before $-t_A^*$ in any PBE. Consider the two possibilities as defined above again.

1. Consider the first possibility.

   If "$v_i^{BR_j} < \sup_{\{x_i\} \in \mathcal{X}} v_i(x_i, X)$ for each i" holds in Assumption 3, then $v_{A,t}(\text{enter})$ is strictly decreasing in t. Hence, candidate A obtains

   $$v_{A,\bar{t}}(\text{enter}) < v_{A,t_A^*}(\text{enter})$$

   if she enters, and she obtains $v_{A,t_A^*}(\text{not})$ if she takes $\bar{\sigma}_A$. Given that $v_{A,t_A^*}(\text{enter}) = v_{A,t_A^*}(\text{not})$, candidate A at $-\bar{t}$ strictly prefers $\bar{\sigma}_A$ to entering.

2. Under the second possibility, (i) given that candidate B does not enter, candidate A weakly prefers not entering and (ii) conditional on the event that candidate B has an opportunity and enters at $-t$, candidate A at $-\bar{t}$ strictly prefers not entering to entering since $v_A^{BR_B} < \hat{v}_{A,\check{t}}$. Hence, candidate A at $-\bar{t}$ strictly prefers $\bar{\sigma}_A$ to entering.

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Finally, if $t_A^* \neq t_B^*$ holds in Assumption 3, then since $t_A^* \neq t_B^*$ implies $t_A^* < t_B^*$, by the continuity of the continuation payoff in time, there exists $\varepsilon > 0$ such that candidate B enters for each time in $\in (-t_A^* - \varepsilon, -t_A^*]$ given $\bar{\sigma}_B$. Hence, if $t_A^* \neq t_B^*$ holds in Assumption 3, then candidate B enters with a positive probability in $(-\bar{t}, -t_A^*)$, which implies that the second possibility happens with positive probability. Therefore, candidate A at $-\bar{t}$ strictly prefers $\bar{\sigma}_A$ to entering. This completes the proof. □

J.4 Proof of Part 1 of Proposition 9

By the continuity of the continuation payoff in time, for times $-t < -\hat{t}_A$ sufficiently close to $-\hat{t}_A$, candidate B enters, and thus we focus on candidate A's incentive at those times. Let

$$\hat{v}_{A,t} := (1 - e^{-\lambda_A t}) \left(\max_{X_A \in \mathcal{X}} v_A(X_A, x_B^*) \right) + e^{-\lambda_A t} v_A(X, x_B^*)$$

be candidate A's payoff when she has not entered and candidate B has at time $-t$. The straightforward algebra shows that $\bar{v}_{A,t}^A(\text{not})$ satisfies

$$
\begin{aligned}
\bar{v}_{A,t}^A(\text{not}) &= \int_0^t \lambda_B e^{-\lambda_B \tau} \hat{v}_{A,t-\tau} d\tau \\
&= \left(e^{-\lambda_A t} - e^{-(\lambda_1 + \lambda_2)t} \right) v_A\left(X, x_B^*\right) + \left(1 - e^{-2\lambda_B t} - 2e^{-\lambda_B t}\right) \max_{X_A \in \mathcal{X}} v_A\left(X_A, x_B^*\right).
\end{aligned}
$$

In contrast, we have

$$v_{A,t}(\text{enter}) = e^{-\lambda_B t} v_A\left(x_A^*, X\right) + \left(1 - e^{-\lambda_B t}\right) v_A^{BR_B}.$$

Hence, $v_{A,t}(\text{enter})$ and $\bar{v}_{A,t}^A(\text{not})$ are differentiable in t. Since $\hat{t}_A$ is the infimum of t with $\bar{v}_{A,t}^A(\text{not}) \leq v_{A,t}(\text{enter})$, we have

$$\left. \frac{d}{dt} \bar{v}_{A,t}^A(\text{not}) \right|_{t=\hat{t}_A} < \left. \frac{d}{dt} v_{A,t}(\text{enter}) \right|_{t=\hat{t}_A}.$$

Consider candidate A's incentive at time $-\hat{t}_A$. For any $\varepsilon > 0$, there are the following three cases (assuming that candidate B enters as soon as she obtains an opportunity):

1. Candidate A has the next opportunity at time $-\bar{t} \in (-t, -(t - \varepsilon)]$. Conditional on this event, since we fix candidate B's strategy at histories in (X, X), candidate A is indifferent between

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entering and not entering.

2. Candidate B has the next opportunity at time $-\bar{t} \in (-t, -(t-\varepsilon)]$. Conditional on this event, candidate A obtains a payoff of $v_A^{BR_B}$ when she enters at $-t$ and a payoff of $\hat{v}_{A,\bar{t}}$ when she does not.

3. No candidate has an opportunity at any time $-\bar{t} \in (-t, -(t-\varepsilon)]$. Conditional on this event, candidate A obtains a payoff of $v_{A,t-\varepsilon}(\text{enter})$ when she enters at $-t$ and a payoff of $\bar{v}_{A,t-\varepsilon}^A(\text{not})$ when she does not.

Since candidate A is indifferent between entering and not entering at time $-\hat{t}_A$, for any $\varepsilon > 0$, we have

$$
\int_{\tau=0}^{\varepsilon} \underbrace{\lambda_B e^{-(\lambda_A+\lambda_B)\tau}}_{\text{Candidate } B \text{ has the next opportunity at time } -t+\tau} \left(v_A^{BR_B} - \hat{v}_{A,t-\tau} \right) d\tau
$$
$$
= e^{-(\lambda_A+\lambda_B)\varepsilon} \left(\bar{v}_{A,t-\varepsilon}^A(\text{not}) - v_{A,t-\varepsilon}(\text{enter}) \right).
$$

Dividing both sides by ε and taking the limit as $\varepsilon \downarrow 0$, we have

$$
v_A^{BR_B} - \hat{v}_{A,\hat{t}_A} = \left(\frac{d}{dt}\bar{v}_{A,t}^A(\text{not})\bigg|_{t=\hat{t}_A} - \frac{d}{dt}v_{A,t}(\text{enter})\bigg|_{t=\hat{t}_A} \right) < 0.
$$

By the continuity of the continuation payoff in time, there exists $\bar{\eta} > 0$ such that, for each $\eta \in [0, \bar{\eta})$, we have

$$
v_A^{BR_B} - \hat{v}_{A,\hat{t}_A+\eta} < 0. \tag{21}
$$

We now consider candidate A's incentive at $-t < -\hat{t}_A$. Similarly to before, there are the following three cases:

1. Candidate A has the next opportunity at time $-\bar{t} \in (-t, -\hat{t}_A]$. Conditional on this event, since we fix candidate B's strategy at histories in (X, X), candidate A is indifferent between entering and not entering.

2. Candidate B has the next opportunity at time $-\bar{t} \in (-t, -\hat{t}_A]$. Conditional on this event, candidate A obtains a payoff of $v_A^{BR_B}$ when she enters at $-t$ and a payoff of $\hat{v}_{A,\bar{t}}$ when she does not.

3. No candidate has an opportunity at any time $-\bar{t} \in (-t, -\hat{t}_A]$. Conditional on this event, candidate A is indifferent between entering and not entering.

Hence, (21) implies that, for $-t \in (-\hat{t}_A - \bar{\eta}, -\hat{t}_A)$, candidate A strictly prefers to enter in any PBE, as desired.

J.5 Proof of Part 2 of Proposition 9

Since candidate A does not enter for each $-t \in [-\hat{t}_B, 0]$, the fact that candidate B strictly prefers to enter at time 0 and becomes indifferent between entering and not entering at $-\hat{t}_B$ implies that he is strictly worse off if candidate A enters than if she does not enter, after candidate B enters:

Lemma 8. $\hat{t}_B \leq \hat{t}_A$ implies $v_B(x_B^*, X) > v_B^{BR_A}$.

Proof. Suppose otherwise. Then, by Assumption 2, we have $v_B(x_B^*, X) = v_B^{BR_A}$ and so we have

$$v_{B,t}(\text{enter}) = v_B(x_B^*, X).$$

At time $-\hat{t}_B$, consider the following three cases:

1. Candidate A has the next opportunity at time $-t \in (-\hat{t}_B, 0]$. Conditional on this event, candidate B obtains a payoff of $v_B^{BR_A} = v_B(x_B^*, X)$ when he enters at $-t$ and a payoff of $\bar{v}_{B,t}^A(\text{not})$ when he does not.

2. Candidate B has the next opportunity at time $-t \in (-\hat{t}_B, 0]$. Conditional on this event, since we fix candidate A's strategy at histories in (X, X), candidate B is indifferent between entering and not entering.

3. No candidate has an opportunity at any time $-\bar{t} \in (-\hat{t}_B, 0]$. Conditional on this event, candidate B strictly prefers to enter since $v_B(x_B^*, X) > v_B(X, X)$.

Hence, candidate B strictly prefers to enter at $-\hat{t}_B$, which is a contradiction. $\qquad \square$

Given this lemma, we are left to show that, at each $-t \in (-\infty, -\hat{t}_B]$, given that no candidate enters for $-\tau \in (-t, -\hat{t}_B)$, each candidate strictly prefers not to enter at $-t < -t_B$.

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On the one hand, if candidate i enters at $-t$, her payoff is $v_{i,t}(\text{enter})$. By Lemma 8, $v_{i,t}(\text{enter}) < v_{i,\hat{t}_B}(\text{enter})$. On the other hand, if she does not enter, then her payoff is $\bar{v}^A_{i,\hat{t}_B}(\text{not})$. Hence, it is indeed uniquely optimal not to enter at $-t$.

K A Proof and Additional Discussions for Section 2.1

This section provides discussions of the valence election campaign model. First, Section K.1 provides a proof of Proposition 2. Next, Section K.2 derives empirical implications of our model. Then, Section K.3 conducts a welfare analysis, comparing our model with that of Aragonès and Palfrey (2002).

K.1 Proof of Proposition 2

Note that Assumptions 1 and 2 in Appendix F are satisfied given $X_i^* = \{1\}$. Moreover, we have $v_i^{BR_j} < \sup_{x_i \in \mathcal{X}} v_i(x_i, X)$ (Assumption 3) and first-mover disadvantage* is satisfied for $i = W$.

Fix a PBE σ arbitrarily. Given candidate i's history $h_i^t = \left(\left(t_i^k, X_i^k\right)_{k=1}^{k_i}, \left(t_j^l, X_j^l\right)_{l=1}^{l_j}, t, z_i \right)$ at $-t$, let $w_t^i\left(\sigma, h_i^t\right)$ be candidate i's continuation payoff at time $-t$ given σ and h_i^t. In addition, let $\theta(h_i^t) = (X_i^{k_i}, X_j^{l_j})$ be the profile of policy sets that are chosen most recently, where we always write S's current policy set first in this proof. Since the most recently chosen policy sets are observable, we have $\theta(h_S^t) = \theta(h_W^t)$. For simpler notation, we write $\theta(h_S^t) = \theta(h_W^t) = \theta\left(h^t\right)$. By Theorem 3, there exists $v_{i,t}\left(\theta\left(h^t\right)\right)$ such that $w_t^i\left(\sigma, h_i^t\right) = v_{i,t}\left(\theta\left(h^t\right)\right)$ in any PBE σ.

From Lemma 1, the following statements are true:

- If $\theta(h^t) = (\{x\}, \{0, 1\})$ with $x \in \{0, 1\}$ and if W can move, then W is indifferent between entering at $x' \in \{0, 1\}$ with $x' \neq x$ and announcing $\{0, 1\}$. S wins if and only if the median voter is located at x.

- If $\theta(h^t) = (\{0, 1\}, \{x\})$ with $x \in \{0, 1\}$ and if S can move, then S enters at x and wins.

Hence, we have

$$v_{S,t}\left(\theta\left(h^t\right)\right) = 1 - (1-p)\,e^{-\lambda t} \text{ if } \theta(h^t) = (\{0, 1\}, \{1\});$$
$$v_{W,t}\left(\theta\left(h^t\right)\right) = (1-p)\,e^{-\lambda t} \text{ if } \theta(h^t) = (\{0, 1\}, \{1\}).$$

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Similarly, we have

$$
\begin{aligned}
v_{S,t}\left(\theta\left(h^t\right)\right) &= 1-p \text{ if } \theta\left(h^t\right) = (\{1\}, \{0,1\}); \\
v_{W,t}\left(\theta\left(h^t\right)\right) &= p \text{ if } \theta\left(h^t\right) = (\{1\}, \{0,1\}).
\end{aligned}
$$

When $-t$ is sufficiently close to the deadline 0, then at any h^t with $\theta(h^t) = (\{0,1\}, \{0,1\})$, the following are true:

- If W can move, then W enters at 1. Note that, since $-t$ is sufficiently close to zero, with a probability close to 1, there is no more opportunity to announce a policy. Hence, $\{1\}$ gives W the payoff close to $1-p$, $\{0\}$ gives W the payoff close to p, and $\{0,1\}$ gives W the payoff close to zero. S wins if and only if the median voter is located at 0.

- If S can move, then S does not enter. Note that, since $-t$ is sufficiently close to zero, with a probability close to 1, there is no more opportunity to announce a policy. Hence, $\{1\}$ gives S the payoff close to $1-p$, $\{0\}$ gives S the payoff close to p, and $\{0,1\}$ gives S the payoff close to 1.

Hence, we are in Case 3 for Theorem 4 (with candidate A being S), and using the notation of Section F.3, we have

$$
\begin{aligned}
\bar{v}^S_{S,t}(\text{not}) &= 1 - (1-p)\,\lambda t e^{-\lambda t}; \\
v_{S,t}(\text{enter}) &= 1 - p;
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{v}^S_{W,t}(\text{not}) &= (1-p)\,\lambda t e^{-\lambda t}; \\
v_{W,t}(\text{enter}) &= (1-p)\,e^{-\lambda t}.
\end{aligned}
$$

Hence, $\hat{t}_S$ and $\hat{t}_W$, whose notation is introduced in Appendix F, are characterized, respectively, by the infimum of $\hat{t}'_S$ and $\hat{t}'_W$ such that the following inequalities hold:

$$
1 - (1-p)\,\lambda \hat{t}'_S e^{-\lambda \hat{t}'_S} \le 1 - p \Leftrightarrow \frac{p}{1-p} \le \lambda \hat{t}'_S e^{-\lambda \hat{t}'_S} \tag{22}
$$

and

$$(1-p)\lambda \hat{t}'_W e^{-\lambda \hat{t}'_W} \geq (1-p) e^{-\lambda \hat{t}'_W} \Leftrightarrow \hat{t}'_W \geq \frac{1}{\lambda}. \tag{23}$$

To fully characterize the candidates' strategies, we examine the following three possible cases.

Case (1): $\frac{p}{1-p} > e^{-1}$. In this case, we have $\frac{1}{\lambda} = \hat{t}_W < \hat{t}_S$. Hence, Proposition 9 ensures that both S and W announce $\{0,1\}$ for all time in $(-t, -t^*)$ with $t^* := \hat{t}_W$. By Lemma 2, we have shown the claims.

Case (2): $\frac{p}{1-p} < e^{-1}$. In this case, we have $\frac{1}{\lambda} = \hat{t}_W > \hat{t}_S$. Moreover, by the implicit function theorem, we have

$$\frac{d\hat{t}_S}{dp} = \frac{\frac{d\left(\frac{p}{1-p}\right)}{dp}}{\frac{d\lambda \hat{t}_S e^{-\lambda \hat{t}_S}}{d\hat{t}_S}} = \frac{1}{(1-p)^2 \lambda e^{-\lambda \hat{t}_S} \left(1 - \lambda \hat{t}_S\right)} > 0. \tag{24}$$

Recall that the definition of $-\hat{t}_S$ implies that, at time $-\hat{t}_S$, S becomes indifferent between entering at 1 and announcing $\{0,1\}$ given the continuation play in which S does not enter and W enters at times in $(-\hat{t}_S, 0]$. The definition implies that this indifference holds in any PBE. By part 1 of Proposition 9, there exists $\bar{\varepsilon} > 0$ such that both S and W strictly prefer entering at 1 for each $-t \in [-\hat{t}_S - \bar{\varepsilon}, \hat{t}_S)$. Therefore, we are in Case 2 for Theorem 4 with $t_0 = -\hat{t}_S - \bar{\varepsilon}$.

We will show that candidate S always enters at 1 for $-t < -\hat{t}_S$. Suppose S always enters at 1 for all time in $(-t, -\hat{t}_S)$. If S announces $\{0,1\}$ at $-t$, there are the following three subcases to consider.

1. If W can move next by $-\hat{t}_S$, then one strategy that W can take is to announce $\{0,1\}$. The following two cases are possible: If S enters at $\{1\}$ by $-\hat{t}_S$, W gets p. If S does not enter by $-\hat{t}_S$, by the definition of $-\hat{t}_S$ (that is, S is indifferent between $\{1\}$ and $\{0,1\}$ at $-\hat{t}_S$), S gets $1-p$ and W gets p. In both cases, W gets at least p. Furthermore, if W can get the first revision opportunity sufficiently close to $-\hat{t}_S$, W gets strictly more than p since W strictly prefers entering at 1 to announcing $\{0,1\}$. Overall, W gets strictly more than p, which means S gets strictly less than $1-p$.

2. If S can move next by $-\hat{t}_S$, S enters and gets $1-p$.

3. If no candidate can move by $-\hat{t}_S$, then by definition, S gets $1-p$.

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Therefore, the payoff from announcing $\{0,1\}$ is strictly less than $1-p$. This implies that it is uniquely optimal for S to enter at 1, as desired. Hence, $t_S = \infty$ in Proposition 7.

We will now examine candidate W's incentives. Since first-mover disadvantage* for W holds, there exists

$$t_W^* > \hat{t}_S \tag{25}$$

such that it is uniquely optimal for W not to enter at $-t < -t_W^*$ and uniquely optimal for W to enter at $-t \in (-t_W^*, 0]$.[7]

Moreover, $t_W^* = \inf\{t > t_0 : \bar{v}_{W,t}(\text{not}) \geq v_{W,t}(\text{enter})\}$ implies[8]

$$(1-p)\,e^{-\lambda t} = \int_0^{t-\hat{t}_S} e^{-2\lambda\tau}\lambda\,(1-p)\,e^{-\lambda(t-\tau)}d\tau + p\left(1 - \int_0^{t-\hat{t}_S} \lambda e^{-2\lambda\tau}d\tau\right)$$

$\Leftrightarrow$

$$e^{-\lambda\left(2t_W^* - \hat{t}_S\right)} = \frac{p}{1-p}\frac{1}{2}\left(1 + e^{-2\lambda\left(t_W^* - \hat{t}_S\right)}\right).$$

Since $\frac{p}{1-p} = \lambda\hat{t}_S e^{-\lambda\hat{t}_S}$ by the definition of $\hat{t}_S$, this inequality is equivalent to

$$e^{-\lambda\left(2t_W^* - \hat{t}_S\right)} = \lambda\hat{t}_S e^{-\lambda\hat{t}_S}\frac{1}{2}\left(1 + e^{-2\lambda\left(t_W^* - \hat{t}_S\right)}\right) \Leftrightarrow e^{-2\lambda t_W^*} = \frac{\frac{1}{2}\lambda\hat{t}_S}{1 - \frac{1}{2}\lambda\hat{t}_S}e^{-2\lambda\hat{t}_S}.$$

Taking the log of both sides and rearranging, we obtain

$$t_W^* = \hat{t}_S - \frac{1}{2\lambda}\log\left(\frac{\frac{1}{2}\lambda\hat{t}_S}{1 - \frac{1}{2}\lambda\hat{t}_S}\right).$$

Hence, we have

$$\frac{dt_W}{dp} = \frac{dt_W^*}{d\hat{t}_S}\frac{d\hat{t}_S}{dp} = \left(1 - \frac{1}{\lambda\hat{t}_S\left(2 - \lambda\hat{t}_S\right)}\right)\frac{d\hat{t}_S}{dp}.$$

Recalling that $\lambda\hat{t}_S \in (0,1)$, we have

$$\sqrt{\lambda\hat{t}_S\left(2 - \lambda\hat{t}_S\right)} < \frac{1}{2}\left(\lambda\hat{t}_S + \left(2 - \lambda\hat{t}_S\right)\right) = 1,$$

---

[7]This notation of t_W^* is introduced in Appendix J.2 .

[8]Recall that $\bar{v}_{W,t}(\text{not})$ denotes W's expected continuation payoff at time $-t$ when he does not enter, assuming that each candidate will enter at times in $(-t, t_0]$ upon receiving an opportunity (see Appendix J.2 for the formal definition).

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and so

$$\frac{1}{\lambda \hat{t}_S \left(2 - \lambda \hat{t}_S\right)} > 1.$$

Therefore, together with (24), we have

$$\text{sign}\frac{dt_W}{dp} = \text{sign}\left(1 - \frac{1}{\lambda \hat{t}_S \left(2 - \lambda \hat{t}_S\right)}\right) \text{sign}\frac{d\hat{t}_S}{dp} = -1. \tag{26}$$

The inequalities (24), (25), and (26) prove part 2(c) of Proposition 2.

Case (3): $\frac{p}{1-p} = e^{-1}$. At time $-t^* = \frac{1}{\lambda}$, for each h^{t^*} with $\theta(h^{t^*}) = (\{0,1\},\{0,1\})$, S is indifferent between "announcing $\{1\}$ and thereby ensuring $1 - p$," and "announcing $\{0,1\}$." At the same time, W is indifferent between announcing $\{1\}$ and $\{0,1\}$.

For $-t < -t^*$, on the one hand, when W can move, his payoff from not entering is at least p since he gets p if S enters at 1 by $-t^*$. If S does not enter by $-t^*$, by the definition of $-t^*$, S gets $1-p$ and W gets p. On the other hand, entering at 1 gives W a payoff of $1-p$ times the probability of S not having any future revision opportunity, which is equal to $(1 - p)e^{-\lambda t} < (1 - p)e^{-\lambda t^*} = p$. Therefore, W strictly prefers not entering.

Given this, S is always indifferent between "announcing $\{1\}$ and thereby ensuring $1 - p$," and "announcing $\{0,1\}$."

K.2 Empirical Implications

In this section, we derive empirical implications of the results from the model of valence election campaign. We see these implications as only suggestive, but as will be seen in the working paper version of this paper (Kamada and Sugaya [2019]), it is possible to enrich the model by incorporating various features (such as heterogenous arrival rates and general utilities from the outcomes). This suggests that, if one wants to conduct empirical research, then it will be possible to extend the model to incorporate more characteristics and to derive testable implications from such a general model, as we do here for the base model.

First, we show that ambiguity is likely when the probability distribution of the median voter's position is close to uniform, that is, when p is close to $\frac{1}{2}$. Specifically, fix a horizon length $T \in (\frac{1}{\lambda}, \infty)$.

Let p^W be the p such that $t_W = T$.[9] By definition, $p^W < \frac{1}{1+e}$. Proposition 2 implies the following:

1. For $p \in (0, \frac{1}{2}) \setminus \{\frac{1}{1+e}\}$, the probabilities of W and S announcing the ambiguous policy are both nondecreasing in p.

2. For $p \in (0, p^W)$, the probability of W announcing the ambiguous policy is constant in p, and that of S announcing the ambiguous policy is strictly increasing in p.

3. For $p \in (p^W, \frac{1}{1+e})$, the probabilities of W and S announcing the ambiguous policy are both strictly increasing in p.

4. For $p \in (\frac{1}{1+e}, \frac{1}{2})$, the probabilities of W and S announcing the ambiguous policy are constant in p.

Hence, roughly, as the position of the median voter becomes more unpredictable, the probability of ambiguous policy announcement at the election date increases. This is consistent with Campbell (1983), who suggests that opinion dispersion has a strong positive effect on the ambiguity in candidates' language.[10]

Next, suppose that there are two candidates A and B, and outside researchers know $p > \frac{1}{1+e}$ but do not know which candidate is strong and which candidate is weak. They have a prior that assigns a positive probability to both candidate A's being strong and candidate B's being strong. If the researchers can observe the campaign phase, the first entrant can be inferred to be weak (and if there is no entrance, then the posterior about valence is the same as the prior). In contrast, if they cannot observe the campaign phase but only the final policy choices by the candidates, then if only one candidate enters, such a candidate can be inferred to be weak. Otherwise, the posterior about valence is the same as the prior.

K.3 Welfare Comparison with the Static Model

As mentioned in Remark 4, conducting a welfare analysis necessitates us to impose some specific assumption about the voter distribution. Here, we assume that there is a single voter. It is then

---

[9]Such p^W exists and is unique due to Proposition 2 2(c) and $t^* = \frac{1}{\lambda}$.

[10]As discussed in footnote 29 of the main text, we have in mind a situation where n voters are independently distributed over $\{0, 1\}$ where the probability on the policy 0 is $q < \frac{1}{2}$. A higher q suggests more option dispersion (a higher standard deviation of the preferred policies among the voters. Campbell (1983) also considers standard deviation), and corresponds to a higher p.

necessary that this voter's ideal policy is 0 with probability p and 1 with probability $1 - p$. We focus on the case in which $p > \frac{1}{1+e}$. Normalize the voter's payoff so that $u(0) - u(1) = 1$. With this normalization, if a candidate $i \in \{S, W\}$ wins and implements a policy $x \in \{0, 1\}$, the payoff of the voter with the ideal policy $y \in \{0, 1\}$ can be written as $\mathbb{I}_{x=y} + \delta \cdot \mathbb{I}_{i=S}$.

Aragonès and Palfrey (2002) consider the one-shot game where each of candidates S and W simultaneously chooses a policy. Here we consider a version of their model adopted to our environment in which the policy space is $\{0, 1\}$. That is, each candidate chooses either 0 or 1, and there is no choice of $\{0, 1\}$.

Since their expected payoffs are represented by the following payoff matrix, the unique mixed-strategy Nash equilibrium is that S takes 0 and 1 with probabilities p and $1 - p$, respectively; and W takes 0 and 1 with probabilities $1 - p$ and p, respectively:

S's policy \ W's policy	0	1
0	$1, 0$	$p, 1 - p$
1	$1 - p, p$	$1, 0$

Given this equilibrium strategy, the expected welfare of the voter is

$$\underbrace{p}_{y=0} \left(\underbrace{p}_{x_S=0} (1 + \delta) + \underbrace{(1-p)^2}_{x_S=1 \text{ and } x_W=0} + \underbrace{p(1-p)\delta}_{x_S=1 \text{ and } x_W=1} \right)$$

$$+ \underbrace{(1-p)}_{y=1} \left(\underbrace{(1-p)}_{x_S=1} (1 + \delta) + \underbrace{p^2}_{x_S=0 \text{ and } x_W=1} + \underbrace{(1-p)p\delta}_{x_S=0 \text{ and } x_W=0} \right)$$

$$= (1 + \delta) \left(1 - p + p^2 \right),$$

where x_i for $i = S, W$ denotes the realized policy choice by candidate i. This expected payoff converges to $\bar{V}(p) := 1 - p + p^2$ in the limit as δ goes to 0.

Next, consider our model of valence election campaign, supposing that T is sufficiently large ($T \geq \frac{1}{\lambda}$). Since $p > \frac{1}{1+e}$, given Proposition 2, in any PBE, W does not enter for each $-t < -t_W = -\frac{1}{\lambda}$, and enters at $x = 1$ for each $-t > -\frac{1}{\lambda}$, while S never enters unless W enters. Hence, (i) with probability $e^{-\lambda \cdot \frac{1}{\lambda}} = e^{-1}$, no candidate enters; (ii) with probability $\int_0^{\frac{1}{\lambda}} \lambda e^{-\lambda s} e^{-\lambda(\frac{1}{\lambda} - s)} ds = e^{-1}$, W enters at policy 1 but S does not enter; and (iii) with probability $1 - 2e^{-1}$, both candidate

enter at policy 1. In the respective cases, (i) if no candidate enters, then the voter's expected payoff is $\frac{1}{2} + \delta$ (recall that we assume that a candidate without specifying her policy takes each policy with probability $\frac{1}{2}$); (ii) if W enters at 1 while S does not enter, then the expected payoff is $1 - p + p\left(\frac{1}{2} + \delta\right)$; and (iii) if both candidates enter at 1, then the expected payoff is $1 - p + \delta$. In total, the expected payoff is

$$e^{-1}\left(\frac{1}{2} + \delta\right) + e^{-1}\left(1 - p + p\left(\frac{1}{2} + \delta\right)\right) + \left(1 - 2e^{-1}\right)(1 - p + \delta)$$
$$= 1 - \frac{1}{2}e^{-1} - p + \frac{3}{2}pe^{-1} + \delta - (1 - p)\,\delta e^{-1}.$$

This expected payoff converges to $\tilde{V}(p) := 1 - \frac{1}{2}e^{-1} - \left(1 - \frac{5}{2}e^{-1}\right)p$ in the limit as δ goes to 0.

Finally, we compare the two expected payoffs.

$$\bar{V}(p) > \tilde{V}(p) \iff 1 - p + p^2 > 1 - \frac{1}{2}e^{-1} - \left(1 - \frac{5}{2}e^{-1}\right)p$$
$$\iff p^2 + \frac{5}{2}pe^{-1} + \frac{1}{2}e^{-1} > 0,$$

which holds for any $p > 0$. Hence, in particular, we obtain $\bar{V}(p) > \tilde{V}(p)$ for $p > \frac{1}{1+e}$.

The above calculation implies that the voter's expected payoff in our model is smaller than under a unique mixed Nash equilibrium model in which each candidate chooses between 0 and 1 as in Aragonès and Palfrey (2002) when $p > \frac{1}{1+e}$, $\delta > 0$ is sufficiently small, and T is sufficiently large. The intuition is that the policy announcement timing game results in a positive correlation between candidates' positions due to S's motive to copy W's policy. This is ex ante not desirable for the median voter who would like to choose a candidate depending on the realization of the median voter's position. In other words, the presence of such correlation implies that the probability that there is a candidate at the median voter's bliss point is small. In contrast, the Nash equilibrium in the static model entails independence of the probability distributions of the candidates' positions (by definition), so it is more likely that the candidates' positions differ, which enables the median voter to pick the candidate with the same position as the bliss point.

K.4 The Calculation of the Cost of the Commitment for Remark 6

The cost of the commitment can be computed as follows. First, S's payoff from the policy announcement timing game, which we calculate by subtracting W's payoff from 1, is:

$$1 - (1-p) \int_0^{\frac{1}{\lambda}} \lambda e^{-\lambda t} e^{\lambda(\frac{1}{\lambda}-t)} dt = 1 - (1-p)e^{-1}.$$

Next, S's payoff from the commitment scenario is $1 - p$. Subtracting the latter from the former, we obtain $p - (1-p)e^{-1}$.

L Proofs for Section 2.2

L.1 Proof of Proposition 3

Part 1: Policy $x^*(X, \mu)$ is a Condorcet winner. To see why, first note that, given the definition of $\mathcal{M}$, $S_i(y, x^*(X, \mu)) < \frac{1}{2}$ for any $y \neq x^*(X, \mu)$ because X is the support of μ and is connected. This implies that $v_i(y, x^*(X, \mu)) = 0$ for any $y \neq x^*(X, \mu)$. This and $v_i(x^*(X, \mu), x^*(X, \mu)) = \frac{1}{2}$ (which holds by assumption) imply that $x^*(X, \mu)$ is a unique best response to $x^*(X, \mu)$. Second, we have $v_i(x^*(X, \mu), X) = 1$ by assumption. Hence, $v_i(x^*(X, \mu), X) \geq v_i(X_i, X)$ for any $X_i \in \mathcal{X}$.

Since the game is constant-sum, Theorem 2 implies that, in any PBE, each candidate enters at $x^*(X, \mu)$ as soon as possible.

Part 2: There exists a function $y : X \to X$ such that $P_i(x, y(x)) < \frac{1}{2}$ for each $x \in X$ for each $i = A, B$. If candidate i has not entered and j has already entered at x, then it is optimal for i to enter at $y(x)$, which gives the highest feasible payoff. If a candidate enters while the other candidate has not yet entered, then she is indifferent among any policy x with $v_i(\{x\}, X) = 1$ (which exists by assumption) since once the other candidate enters later, she will lose for sure. Therefore, Assumptions 1-3 and first-mover disadvantage* for i in Appendix F are satisfied for each $i \in \{A, B\}$. Moreover, each candidate has a strict incentive to enter at $t = 0$. Hence, we have Case 2 with $t_0 = 0$ for Theorem 4. Hence, Theorem 4 implies that, for each i, there exists t_i such that candidate i enters at all times $-t \in (-t_i, 0]$ and does not enter at all times $-t \in (-\infty, -t_i)$.

In addition, t_i^* in Appendix F is calculated as follows: On the one hand, i's expected payoff of entering is the probability that the other candidate will not have an opportunity to enter. That is,

$v_{i,t}(\text{enter}) = e^{-\lambda_j t}$. On the other hand, supposing that each player enters at every time $-\tau \in (-t, 0]$, we have

$$\bar{v}_{i,t}(\text{not}) = \left(\int_0^t \underbrace{e^{-(\lambda_i + \lambda_j)\tau}}_{\text{no one moves until time } -(t-\tau)} \quad (\underbrace{\lambda_i d\tau}_{i \text{ moves at time } -(t-\tau)}) \quad \underbrace{e^{-\lambda_j(t-\tau)}}_{j \text{ does not move in } (-(t-\tau), 0]} \right)$$

$$+ \left(\int_0^t \underbrace{e^{-(\lambda_i + \lambda_j)\tau}}_{\text{no one moves until time } -(t-\tau)} \quad (\underbrace{\lambda_j d\tau}_{j \text{ moves at time } -(t-\tau)})(\underbrace{1 - e^{-\lambda_i(t-\tau)}}_{i \text{ can move in } (-(t-\tau), 0]}) \right) + \underbrace{e^{-(\lambda_i+\lambda_j)t}}_{\text{no one moves until time } 0} \frac{1}{2}.$$

Hence, t_i^* is characterized by $f_i(t_i^*) = 0$ with

$$f_i(t) := -e^{-\lambda_i t} + \frac{\lambda_j}{\lambda_i + \lambda_j}(1 - e^{-(\lambda_i + \lambda_j)t}) + e^{-(\lambda_i + \lambda_j)t}\frac{1}{2}. \tag{27}$$

Differentiating $f_i(t)$, we get

$$f_i'(t) = \lambda_i(e^{-\lambda_i t} - e^{-(\lambda_i + \lambda_j)t}\frac{1}{2}) + \lambda_j e^{-(\lambda_i + \lambda_j)t}\frac{1}{2} > 0.$$

Since $f_i(t)$ is $-\frac{1}{2}$ at $t = 0$, continuous and strictly increasing in t, and approaches $\frac{\lambda_j}{\lambda_i + \lambda_j} > 0$ as $t \to \infty$, there exists a unique t such that $f_i(t) = 0$. The cutoff t_i^* is such t.

To prove that $\text{sign}\left(t_i^* - t_j^*\right) = \text{sign}(\lambda_i - \lambda_j)$, we first show that $f_i(t) < f_j(t)$ for each $t > 0$. To see this, suppose $\lambda_i > \lambda_j$. Given

$$f_i(t) := -e^{-\lambda_i t} + \frac{\lambda_j}{\lambda_i + \lambda_j}(1 - e^{-(\lambda_i + \lambda_j)t}) + e^{-(\lambda_i + \lambda_j)t}\frac{1}{2},$$

we have

$$\frac{\partial\left(f_i(t) - f_j(t)\right)}{\partial t} = \frac{e^{-t(\lambda_i + \lambda_j)}\left(-\lambda_i + \lambda_j + \lambda_i e^{\lambda_j t} - \lambda_j e^{\lambda_i t}\right)}{2(\lambda_i + \lambda_j)^2}.$$

Hence, the sign of $\frac{\partial(f_i(t) - f_j(t))}{\partial t}$ is equal to the sign of

$$g(t) = -\lambda_i + \lambda_j + \lambda_i e^{\lambda_j t} - \lambda_j e^{\lambda_i t}.$$

Since

$$\frac{\partial g(t)}{\partial t} = \lambda_i \lambda_j \left(-e^{\lambda_i t} + e^{\lambda_j t}\right) < 0 \text{ given } \lambda_i > \lambda_j,$$

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we have

$$g(t) < g(0) = 0 \text{ for each } t > 0.$$

Hence, $\frac{\partial(f_i(t) - f_j(t))}{\partial t} < 0$ for each $t > 0$. Since $f_i(0) = f_j(0)$, we have $f_i(t) < f_j(t)$ for each $t > 0$.

Now, given that $f_i(t) < f_j(t)$ for each $t > 0$, at time $-t_j^*$ such that $f_j(t_j^*) = 0$, we have $f_i(t_j^*) < 0$. This inequality implies that $t_i^* > t_j^*$ given $\lambda_i > \lambda_j$ because $f'(t) > 0$.

L.2 Proof of Proposition 4

Fix a PBE σ and consider A's deviation to σ_A', defined as follows:

1. At any time in $(-\infty, 0]$, under histories such that B has announced a policy x', A announces a policy x'' such that $P_A(x'', x') = 1$ with probability one.[11]

2. Under the histories such that B has only announced X, the following holds.

   (a) At times in $(-\infty, -t_B^*)$, A keeps being ambiguous (announces X).

   (b) At times in $(-t_B^*, 0]$, she uses a mixed action as follows. With probability $\frac{\lambda_B}{\lambda_A}$ (which is strictly between 0 and 1 as $0 < \lambda_B < \lambda_A$), she announces a policy $\bar{x}$ such that $P_A(\bar{x}, X) = 1$.[12] With the remaining probability, she keeps being ambiguous (announces X).

First, we show that candidate A obtains a strictly higher payoff than candidate B under (σ_A', σ_B). To see this, notice that the probability that A enters when B is still announcing X and the probability that B enters when A is still announcing X are the same under this strategy profile. However, the conditional probability that B enters after A's entry is strictly lower than the conditional probability that A enters after B's entry. This is because, as specified in item 1 above, A can move "more quickly" than B can.

Formally, candidate i obtains payoff 1 if and only if either (i) she is the only candidate who enters, or (ii) $-i$ enters first and then i enters. She obtains payoff $\frac{1}{2}$ if no candidate enters. In all other cases, she obtains payoff 0. Hence, the payoff difference between A and B arises solely from the probability of the event in which payoff 1 is obtained. As explained above, this probability is

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[11]Such x'' exists by assumption.
[12]Such $\bar{x}$ exists by assumption.

strictly higher for candidate A than candidate B. Therefore, A's payoff is strictly greater than B's payoff under (σ'_A, σ_B), i.e.,

$$u_A(\sigma'_A, \sigma_B) > u_B(\sigma'_A, \sigma_B), \tag{28}$$

where the notation is from Section 3 and we drop references to the belief and the initial history (on which the payoffs condition).

Second, notice that candidate A's payoff under σ is no less than her payoff under (σ'_A, σ_B). Since the election is constant-sum, this means that candidate B's payoff under σ is no more than his payoff under (σ'_A, σ_B). Hence we have the following relationships:

$$u_A(\sigma) \geq u_A(\sigma'_A, \sigma_B) \quad \text{and} \quad u_B(\sigma'_A, \sigma_B) \geq u_B(\sigma). \tag{29}$$

Combining (28) and (29) implies $u_A(\sigma) > u_B(\sigma)$, i.e., under σ, candidate A receives a strictly higher payoff than candidate B does.

M Proofs for Section 2.3

M.1 Payoff Matrix

Defining $\bar{X} \equiv \{1, 0\}$, candidate S's payoff matrix of the election (we omit candidate W's payoff since the game is constant-sum) is given by

$S\backslash W$	$\bar{X}, 0$	$\bar{X}, 1$	$\bar{X}, \bar{X}$	$1, 0$	$1, 1$
$\bar{X}, 0$	1	p_2	p_2	p_1	p_2
$\bar{X}, 1$	$1 - p_2$	1	$1 - p_2$	$1 - p_2$	p_1
$\bar{X}, \bar{X}$	$1 - p_2$	p_2	1	$1 - (1 - p_1)\, p_2$	$1 - (1 - p_1)\,(1 - p_2)$
$1, 0$	$1 - p_1$	p_2	$(1 - p_1)\, p_2$	1	p_2
$1, 1$	$1 - p_2$	$1 - p_1$	$(1 - p_1)\,(1 - p_2)$	$1 - p_2$	1
$1, \bar{X}$	$1 - p_1 p_2$	$1 - p_1\,(1 - p_2)$	$1 - p_1$	$1 - p_2$	p_2
$0, 0$	p_1	p_2	$p_1 p_2$	p_1	$1 - (1 - p_1)\,(1 - p_2)$
$0, 1$	$1 - p_2$	p_1	$p_1\,(1 - p_2)$	$1 - (1 - p_1)\, p_2$	p_1
$0, \bar{X}$	$1 - (1 - p_1)\, p_2$	$1 - (1 - p_1)\,(1 - p_2)$	p_1	p_1	p_1

$S\backslash W$	$1, \bar{X}$	$0, 0$	$0, 1$	$0, \bar{X}$
$\bar{X}, 0$	$1 - (1 - p_1)(1 - p_2)$	$1 - p_1$	p_2	$1 - p_1(1 - p_2)$
$\bar{X}, 1$	$1 - (1 - p_1) p_2$	$1 - p_2$	$1 - p_1$	$1 - p_1 p_2$
$\bar{X}, \bar{X}$	p_1	$1 - p_1 p_2$	$1 - p_1(1 - p_2)$	$1 - p_1$
$1, 0$	p_2	$1 - p_1$	$1 - p_1(1 - p_2)$	$1 - p_1$
$1, 1$	$1 - p_2$	$1 - p_1 p_2$	$1 - p_1$	$1 - p_1$
$1, \bar{X}$	1	$1 - p_1$	$1 - p_1$	$1 - p_1$
$0, 0$	p_1	1	p_2	p_2
$0, 1$	p_1	$1 - p_2$	1	$1 - p_2$
$0, \bar{X}$	p_1	$1 - p_2$	p_2	1

M.2 Proofs

We first derive properties that hold in a general constant-sum game.

Definition 7. Let v_i be a utility function of the election. Given θ_j, announcement θ_i **dominates** announcement $\tilde{\theta}_i$ **both in payoffs and flexibility** if the following conditions hold:

1. θ_i is strictly better than $\tilde{\theta}_i$ if j stays at θ_j, i.e., $v_i(\theta_i, \theta_j) > v_i\left(\tilde{\theta}_i, \theta_j\right)$.

2. θ_i is weakly better than $\tilde{\theta}_i$ if j moves, i.e., the following hold.

    (a) If $\tilde{\theta}_i$ is not a singleton, $v_i\left(\theta_i, \tilde{\theta}_j\right) \geq v_i\left(\tilde{\theta}_i, \tilde{\theta}_j\right)$ for each $\tilde{\theta}_j \subseteq \theta_j$.

    (b) If $\tilde{\theta}_i$ is a singleton, $\inf_{\tilde{\theta}_j \subseteq \theta_j} v_i\left(\theta_i, \tilde{\theta}_j\right) \geq \inf_{\hat{\theta}_j \subseteq \theta_j} v_i(\tilde{\theta}_i, \hat{\theta}_j)$.

3. (iii) θ_i has more flexibility than $\tilde{\theta}_i$, i.e., $\tilde{\theta}_i \subseteq \theta_i$.

We say announcement θ_i **weakly dominates** announcement $\tilde{\theta}_i$ **both in payoffs and flexibility** given θ_j if the strict inequality in (i) of Definition 7 is replaced with the weak inequality.

Given that (v_i, v_j) is constant-sum, player i's value at time $-t$ given the current policy announcement (θ_i, θ_j), which we denote by $v_{i,t}(\theta_i, \theta_j)$, is well defined (i..e, constant across all PBE) by the minimax theorem and Theorem 3.[13] The following lemma uses the constant-sum nature to

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[13] Gensbittel et al. (2017) show that the minimax theorem extends to revision games with finite actions and payoffs. Since their proof uses results proven in a more general environment of stochastic games (Lovo and Tomala [2015]), it can be easily extended to our case in which the set of available actions can vary depending on the history of play.

show that announcing a policy that is weakly dominated both in payoffs and flexibility does not increase a candidate's payoff.

Lemma 9. *For each i, j, θ_j, θ_i, and $\tilde{\theta}_i$, suppose candidate j currently takes θ_j, and given θ_j, announcement θ_i weakly dominates announcement $\tilde{\theta}_i$ both in payoffs and flexibility. Then, $v_{i,t}(\theta_i, \theta_j) \geq v_{i,t}\left(\tilde{\theta}_i, \theta_j\right)$ for each $-t \in (\infty, 0]$.*

Proof. Fix θ_j, θ_i, and $\tilde{\theta}_i$. Let Σ_k be the set of all strategies of candidate k. Given the minimax theorem, for each $(\hat{\theta}_i, \hat{\theta}_j) \in \mathcal{X}_i \times \mathcal{X}_j$, we have

$$
\begin{aligned}
v_{i,t}(\hat{\theta}_i, \hat{\theta}_j) &= \min_{\sigma_j \in \Sigma_j} \max_{\sigma_i \in \Sigma_i} \mathbb{E}\left[v_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \hat{\theta}_i, \theta_{j,T} = \hat{\theta}_j\right] \\
&= \max_{\sigma_i \in \Sigma_i} \min_{\sigma_j \in \Sigma_j} \mathbb{E}\left[v_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \hat{\theta}_i, \theta_{j,T} = \hat{\theta}_j\right].
\end{aligned}
$$

Let $\bar{\Sigma}_i$ be the set of strategies such that player i never enters at $\hat{\theta}_i$ such that $\hat{\theta}_i \neq \theta_i$ and $\hat{\theta}_i \not\subseteq \tilde{\theta}_i$. Intuitively, candidate i commits not to use the advantage of θ_i in flexibility compared to $\tilde{\theta}_i$. Moreover, let $\hat{v}_i : \mathcal{X}_i \times \mathcal{X}_j$ be the utility function such that $\hat{v}_i(\theta_i, \hat{\theta}_j) = v_i(\tilde{\theta}_i, \hat{\theta}_j)$ for each $\hat{\theta}_j$, and $\hat{v}_i(\hat{\theta}_i, \tilde{\theta}_j) = v_i(\hat{\theta}_i, \tilde{\theta}_j)$ for each $\tilde{\theta}_j$ and $\hat{\theta}_i \neq \theta_i$. Intuitively, we cancel out the payoff advantage of θ_i compared to $\tilde{\theta}_i$.

Restricting candidate i's strategy to $\bar{\Sigma}_i$ does not increase candidate i's minimax payoff. Given that candidate i takes a strategy in $\bar{\Sigma}_i$, replacing v_i with $\hat{v}_i$ reduces candidate i's minimax payoff. Hence,

$$
\begin{aligned}
v_{i,t}(\theta_i, \theta_j) &\geq \min_{\sigma_j \in \Sigma_j} \max_{\sigma_i \in \bar{\Sigma}_i} \mathbb{E}\left[\hat{v}_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \theta_i, \theta_{j,T} = \theta_j\right] \\
&= \max_{\sigma_i \in \bar{\Sigma}_i} \min_{\sigma_j \in \Sigma_j} \mathbb{E}\left[\hat{v}_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \theta_i, \theta_{j,T} = \theta_j\right].
\end{aligned}
$$

Given $\hat{v}_i$, for each $\hat{\theta}_j$, player i's final payoff $\hat{v}_i$ given $(\tilde{\theta}_i, \hat{\theta}_j)$ and that given $(\theta_i, \hat{\theta}_j)$ are the same. Moreover, given $\bar{\Sigma}_i$, player i never takes $\hat{\theta}_i \not\subseteq \tilde{\theta}_i$ given θ_i. Hence, the situation is as if the current announcement is $(\tilde{\theta}_i, \theta_j)$ and the utility function is v_i. That is,

$$
\begin{aligned}
&\max_{\sigma_i \in \bar{\Sigma}_i} \min_{\sigma_j \in \Sigma_j} \mathbb{E}\left[\hat{v}_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \theta_i, \theta_{j,T} = \theta_j\right] \\
&= \max_{\sigma_i \in \bar{\Sigma}_i} \min_{\sigma_j \in \Sigma_j} \mathbb{E}\left[v_i(\theta_{i,0}, \theta_{j,0})|T = t, \theta_{i,T} = \tilde{\theta}_i, \theta_{j,T} = \theta_j\right].
\end{aligned}
$$

Combining the two displayed equations implies the result. $\qquad\square$

Similarly, we can show that taking a strictly dominated announcement (both in payoffs and flexibility) reduces a candidate's payoff.

Lemma 10. *For each i, j, θ_j, θ_i, and $\tilde{\theta}_i$, suppose candidate j currently takes θ_j, and given θ_j, announcement θ_i dominates announcement $\tilde{\theta}_i$ both in payoffs and flexibility. Then, $v_{i,t}(\theta_i, \theta_j) > v_{i,t}(\tilde{\theta}_i, \theta_j)$ for each t.*

Proof. Suppose the result holds for all $t' < t$. Fix $\varepsilon > 0$ arbitrarily. Suppose that at time $-\tilde{t} \in (-t-\varepsilon, -t]$, i obtains an opportunity. Consider the following four events:

1. Between $(-\tilde{t}, -t]$, only candidate i has an opportunity. In this case, the inductive hypothesis implies that announcing θ_i is strictly better.

2. Between $(-\tilde{t}, -t]$, only candidate j has an opportunity. Let $-t'$ be the timing at which candidate j obtains an opportunity. If i entered at θ_i at time $-\tilde{t}$, candidate j takes $\hat{\theta}_j$ to achieve $\min_{\hat{\theta}_j \subseteq \theta_j} v_{i,t'}(\theta_i, \hat{\theta}_j)$, and if i entered at $\tilde{\theta}_i$ at time $-\tilde{t}$, candidate j takes $\hat{\theta}_j$ to achieve $\min_{\hat{\theta}_j \subseteq \theta_j} v_{i,t'}(\tilde{\theta}_i, \hat{\theta}_j)$.

   Note that (ii) and (iii) of the definition of the domination given θ_j implies that, whenever θ_i weakly dominates $\tilde{\theta}_i$ given each θ_j, then θ_i weakly dominates $\tilde{\theta}_i$ given each $\hat{\theta}_j \subseteq \theta_j$. Hence, Lemma 9 implies $v_{i,t'}(\tilde{\theta}_i, \hat{\theta}_j) \leq v_{i,t'}(\theta_i, \hat{\theta}_j)$ for each $\hat{\theta}_j \subseteq \theta_j$. Hence, we have $\min_{\hat{\theta}_j \subseteq \theta_j} v_{i,t'}(\theta_i, \hat{\theta}_j) \geq \min_{\hat{\theta}_j \subseteq \theta_j} v_{i,t'}(\tilde{\theta}_i, \hat{\theta}_j)$.

3. No candidate obtains an opportunity. In this case, the inductive hypothesis implies that announcing θ_i is strictly better.

4. Other cases. The likelihood of this event compared to the first three converges to zero as $\varepsilon \to 0$.

Hence, there exists $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$, for each $\tilde{t} \in [t, t+\varepsilon)$, we have $v_{i,\tilde{t}}(\theta_i, \theta_j) > v_{i,\tilde{t}}(\tilde{\theta}_i, \theta_j)$. By continuous-time backward induction, the result follows. $\qquad\square$

M.3 Game with Valence: Proof of Proposition 5

We first note that the weak candidate never fully specifies his policy if the strong candidate's current policy is $(\bar{X}, \bar{X})$:

Lemma 11. *Under any PBE, candidate W never enters at any $(x_1, x_2) \in \{0, 1\} \times \{0, 1\}$ when the current policy set profile is (θ_S, θ_W) with $\theta_S = (\bar{X}, \bar{X})$ and $\theta_W \subseteq \{0, 1\} \times \{0, 1\}$ satisfying $\theta_W \notin \{0, 1\} \times \{0, 1\}$.*

Proof. For candidate W, $(x_1, \bar{X})$ and $(\bar{X}, x_2)$ dominate (x_1, x_2) both in payoffs and flexibility. $\square$

M.3.1 Values after Some Candidate Enters

We first narrow down the set of policies to which S may enter when W has not entered:

Lemma 12. *Under any PBE, candidate S enters at $(1, \bar{X})$ or stays at $(\bar{X}, \bar{X})$ when the current policy set profile is $((\bar{X}, \bar{X}), (\bar{X}, \bar{X}))$. Moreover, her value of entering at $(1, \bar{X})$ is given by*

$$v_{S,t}\left((1, \bar{X}), (\bar{X}, \bar{X})\right) = \begin{cases} 1 - e^{-\lambda t}\left(p_1 + (1 - p_2)\lambda t\right) & \text{if } t \leq \frac{1}{\lambda}\frac{1 - p_1 - p_2}{1 - p_2} \\ 1 - (1 - p_2)e^{-1 + \frac{p_1}{1 - p_2}} & \text{if } t \geq \frac{1}{\lambda}\frac{1 - p_1 - p_2}{1 - p_2} \end{cases}. \tag{30}$$

The continuation play after S's entering at $(1, \bar{X})$ is such that (i) candidate S stays at $(1, \bar{X})$ for each $-t \in (-\infty, 0]$ if W stays at $(\bar{X}, \bar{X})$; (ii) candidate W stays at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -\frac{1}{\lambda}\frac{1 - p_1 - p_2}{1 - p_2})$ and enters at $(1, 1)$ for each $-t \in (-\frac{1}{\lambda}\frac{1 - p_1 - p_2}{1 - p_2}, 0]$; and (iii) candidate S enters at $(1, 1)$ as soon as possible after candidate W enters at $(1, 1)$.

Proof. Fix a PBE. First, for each policy $x \in \{0, 1\}$, given $\theta_S = (x, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$, S does not enter at $(x, 0)$ or $(x, 1)$ since $(x, \bar{X})$ strictly dominates $(x, 0)$ and $(x, 1)$ both in payoffs and flexibility. Similarly, for each policy $x \in \{0, 1\}$, given $\theta_S = (\bar{X}, x)$ and $\theta_W = (\bar{X}, \bar{X})$, S does not enter at $(0, x)$ or $(1, x)$.

Second, for each policy $x \in \{0, 1\}$, given $\theta_S = (x, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$, candidate W does not enter at $(x, \bar{X})$, $(\bar{X}, 0)$, $(\bar{X}, 1)$, $(1 - x, 0)$, or $(1 - x, 1)$ since they are dominated by $(\bar{X}, \bar{X})$ both in payoffs and flexibility. Similarly, given $\theta_S = (x, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$, candidate W does not enter at $(x, 0)$ since entering at $(x, 1)$ gives him a strictly better payoff if candidate S cannot move and both give him 0 if candidate S has another opportunity. In addition, S's value given

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$\theta_S = (x, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$ is calculated by assuming that W never enters at $(1 - x, \bar{X})$ as long as S stays at $(x, \bar{X})$ since $(1 - x, \bar{X})$ is weakly dominated by $(\bar{X}, \bar{X})$ both in payoffs and flexibility. For the same reasons as the case given $\theta_S = (x, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$, for each policy $x \in \{0, 1\}$, given $\theta_S = (\bar{X}, x)$ and $\theta_W = (\bar{X}, \bar{X})$, candidate W does not enter at $(\bar{X}, x)$, $(0, 1 - x)$, $(1, 1 - x)$, $(0, \bar{X})$, $(1, \bar{X})$, or $(0, x)$. In addition, S's value given $\theta_S = (\bar{X}, x)$ and $\theta_W = (\bar{X}, \bar{X})$ is calculated by assuming that W never enters at $(\bar{X}, 1 - x)$ as long as S stays at $(\bar{X}, x)$.

Given these results, once candidate S enters at $(1, \bar{X})$, on the path of play of the fixed PBE, there are four possibilities for the policy set profile at time 0, where the associated payoff for candidate S can be computed as follows:

$S \backslash W$	$(\bar{X}, \bar{X})$	$(1, 1)$
$(1, \bar{X})$	$1 - p_1$	p_2
$(1, 1)$	$(1 - p_1)(1 - p_2)$	1

Given this payoff matrix, near the deadline, candidate W enters at $(1, 1)$ and candidate S stays at $(1, \bar{X})$. Given this continuation strategy, candidate S weakly prefers staying at $(1, \bar{X})$ to entering at $(1, 1)$ if

$$e^{-\lambda t}(1 - p_1) + \int \lambda e^{-\lambda x} \left(e^{-\lambda(t-x)} p_2 + \left(1 - e^{-\lambda(t-x)}\right) 1 \right) dx \geq (1 - p_1)(1 - p_2) \qquad (31)$$

$\Leftrightarrow$

$$p_1 + p_2 - p_1 p_2 - e^{-\lambda t}(p_1 + \lambda t(1 + p_2)) \geq 0, \qquad (32)$$

and candidate W weakly prefers entering at $(1, 1)$ to staying at $(\bar{X}, \bar{X})$ if

$$e^{-\lambda t}(1 - p_2) \geq e^{-\lambda t} p_1 + \int \lambda e^{-\lambda x} \left(e^{-\lambda(t-x)}(1 - p_2) \right) dx$$

$\Leftrightarrow$

$$(1 - \lambda t)(1 - q_2) - q_1 \geq 0. \qquad (33)$$

Solving (33) for t (the solution is $\frac{1}{\lambda} \frac{1 - p_1 - p_2}{1 - p_2}$) and substituting the solution into (32), we can show that the left-hand side of (32) is strictly positive.

With this, as in the one-issue case, we can show that candidate S stays at $(1, \bar{X})$ for each

$-t \in (-\infty, 0]$ and W stays at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -\frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2})$ and enters at $(1,1)$ for each $-t \in (-\frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2}, 0]$. Therefore, when the current policy set profile is $((\bar{X}, \bar{X}), (\bar{X}, \bar{X}))$, the value of entering at $(1, \bar{X})$ for candidate S is given by (30). The fact that candidate S strictly prefers staying at $(1, \bar{X})$ given $((1, \bar{X}), (\bar{X}, \bar{X}))$ implies that it is suboptimal to enter at $(1,1)$ or $(1,0)$ given $((\bar{X}, \bar{X}), (\bar{X}, \bar{X}))$.

So far, we have assumed that W never enters at $(0, \bar{X})$ given $\theta_S = (1, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$. This is without loss when calculating the candidates' values since such behavior is weakly dominated both in payoffs and flexibility. Now we verify that this is the unique behavior in any PBE. For each $X_S \in \{0, 1, \bar{X}\}$ and $X_W \in \{0, 1, \bar{X}\}$, the payoff profile from $((1, X_S), (0, X_W))$ is $(1 - p_1, p_1)$. Note that W's equilibrium value is strictly greater than p_1 since p_1 is the payoff that W can obtain by simply staying at $(\bar{X}, \bar{X})$ given S's strategy and in equilibrium W takes a strictly better strategy than just staying at $(\bar{X}, \bar{X})$ when $-t$ is close to 0. Hence, given $\theta_S = (1, \bar{X})$ and $\theta_W = (\bar{X}, \bar{X})$, W never enters at $(0, \bar{X})$.

We follow the same procedure to calculate S's values when she enters at $(0, \bar{X})$, $(\bar{X}, 1)$, or $(\bar{X}, 0)$. A straightforward algebra implies that entering at $(1, \bar{X})$ achieves the uniquely highest payoff for S. $\qquad\square$

We next narrow down the set of policies to which W may enter when S has not entered. To this end, we first specify the continuation play after W enters at $(1, \bar{X})$:

Lemma 13. *On the path of play of any PBE, candidate S enters at $(1, \bar{X})$ at the history such that the current policy set profile is $((\bar{X}, \bar{X}), (1, \bar{X}))$ and the current time is $-t \in (-\infty, 0]$. Once candidate S enters at $(1, \bar{X})$, the following hold.*

1. *Candidate W stays at $(1, \bar{X})$ for each $-t \in (-\infty, -\frac{1}{\lambda})$ and enters at $(1,1)$ for each $-t \in (-\frac{1}{\lambda}, 0]$.*

2. *Candidate S stays at $(1, \bar{X})$ until candidate W enters at $(1,1)$, and enters at $(1,1)$ as soon as possible once W enters at $(1,1)$.*

Given this continuation play, candidate S's value of candidate W entering at $\left(1, \bar{X}\right)$ is

$$v_{S,t}\left(\left(\bar{X}, \bar{X}\right), \left(1, \bar{X}\right)\right) = \begin{cases} e^{-\lambda t}p_1 + 1 - e^{-\lambda t} + \frac{1}{2}\left(1 - p_2\right)t^2\lambda^2 e^{-\lambda t} & t \leq \frac{1}{\lambda}, \\ e^{-\lambda t}p_1 + \left(1 - e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\right)\left(1 - e^{-1}\left(1 - p_2\right)\right) & t \geq \frac{1}{\lambda}. \\ +e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\left(1 - e^{-1} + \frac{1}{2}\left(1 - p_2\right)e^{-1}\right) \end{cases} \tag{34}$$

Proof. Given $\theta_W = \left(1, \bar{X}\right)$ and $\theta_S = \left(\bar{X}, \bar{X}\right)$, candidate W stays until candidate S moves by the proof of Lemma 11.

In addition, S's value given $\theta_W = \left(1, \bar{X}\right)$ and $\theta_S = \left(\bar{X}, \bar{X}\right)$ can be calculated by assuming that S never enters at $\left(0, \bar{X}\right)$, $(0,0)$, or $(0,1)$ as long as W stays at $\left(1, \bar{X}\right)$. This is because $\left(0, \bar{X}\right)$ is weakly dominated by $\left(\bar{X}, \bar{X}\right)$ both in payoffs and flexibility, and $(0,0)$ and $(0,1)$ are dominated by $\left(\bar{X}, \bar{X}\right)$ both in payoffs and flexibility. Similarly, given $\theta_W = \left(1, \bar{X}\right)$, candidate S never enters at $(1,0)$ since entering at $(1,1)$ gives her a better payoff given candidate W's best response in the continuation play.

If candidate S enters at $\left(1, \bar{X}\right)$, she obtains the same payoff as in the one-issue valence election with $p = p_2$. Since we assume $p_2 \geq \frac{1}{1+e}$, candidate S's payoff of entering at $\left(1, \bar{X}\right)$ is

$$\begin{cases} 1 - \lambda t e^{-\lambda t}\left(1 - p_2\right) & \text{if } t \leq \frac{1}{\lambda}, \\ 1 - e^{-1}\left(1 - p_2\right) & \text{otherwise.} \end{cases}$$

Hence, there exists $\bar{t} > 0$ such that, for all times $-t \in \left(-\bar{t}, 0\right]$, it is optimal to enter at $\left(1, \bar{X}\right)$.[14] Given this continuation strategy, the payoff of staying at $\left(\bar{X}, \bar{X}\right)$ at $-t \geq -\frac{1}{\lambda}$ is

$$e^{-\lambda t}p_1 + \int \lambda e^{-\lambda x}\left(1 - \lambda\left(t - x\right)e^{-\lambda(t-x)}\left(1 - p_2\right)\right)dx.$$

The incentive for S to entering at $\left(1, \bar{X}\right)$ at $-t \geq -\frac{1}{\lambda}$ is

$$1 - \lambda t e^{-\lambda t}\left(1 - p_2\right) \geq e^{-\lambda t}p_1 + \int \lambda e^{-\lambda x}\left(1 - \lambda\left(t - x\right)e^{-\lambda(t-x)}\left(1 - p_2\right)\right)dx,$$

or

$$2 - 2p_1 - \left(1 - p_2\right)\left(2 - \lambda t\right)\lambda t \geq 0.$$

---

[14] In fact, the following proof shows that we can take $\bar{t} = \infty$.

Since

$$2 - 2p_1 - (1 - p_2)(2 - \lambda t)\lambda t \geq 2 - 2p_1 - (1 - p_2) = 1 - 2p_1 + p_2 > 0,$$

candidate S enters at $(1, \bar{X})$ at $-t \in [-\frac{1}{\lambda}, 0]$. Since candidate W does not move from $(1, \bar{X})$ given $\theta_S = (1, \bar{X})$ and $\theta_W = (1, \bar{X})$ or given $\theta_S = (\bar{X}, \bar{X})$ and $\theta_W = (1, \bar{X})$ for each $-t \in (-\infty, -\frac{1}{\lambda})$, this inequality also implies that it is optimal for candidate S to enter at $(1, \bar{X})$ as soon as possible for each $-t \in (-\infty, -\frac{1}{\lambda})$. Hence, candidate S's payoff of candidate W entering at $(1, \bar{X})$ is given by (34).

Finally, we show that S will not enter at $(0, \bar{X})$. To see this, note that once S enters at $(0, \bar{X})$, since the candidates are taking a static strict best response to each other (among feasible actions given the current announcements), by continuous time backward induction, no candidate moves further (W stays at $(1, \bar{X})$ and S stays at $(0, \bar{X})$). Hence, S's payoff is p_1 if she enters at $(0, \bar{X})$. Since the value characterized by (34) is strictly greater than p_1, S does not enter at $(0, \bar{X})$.

□

The next lemma characterizes the continuation play after W enters at $(\bar{X}, 1)$.

Lemma 14. *On the path of play of any PBE, candidate S enters at $(\bar{X}, 1)$ at any history such that the current policy set profile is $((\bar{X}, \bar{X}), (\bar{X}, 1))$. Once candidate S enters at $(\bar{X}, 1)$, the following hold:*

1. *Candidate W stays at $(\bar{X}, 1)$ for each $-t \in (-\infty, -\frac{1}{\lambda})$ and enters at $(1, 1)$ for each $-t \in (-\frac{1}{\lambda}, 0]$.*

2. *Candidate S stays at $(\bar{X}, 1)$ for each $-t \in (-\infty, 0]$ until candidate W enters at $(1, 1)$. Once W enters at $(1, 1)$, S enters at $(1, 1)$ as soon as possible.*

Given this continuation play, candidate S's value of candidate W entering at $(1, \bar{X})$ is

$$v_{S,t}(\bar{X}, \bar{X}, \bar{X}, 1) = \begin{cases} e^{-\lambda t}p_2 + 1 - e^{-\lambda t} - \frac{1}{2}(1 - p_1)t^2\lambda^2 e^{-\lambda t} & t \leq \frac{1}{\lambda}, \\ e^{-\lambda t}p_2 + \left(1 - e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\right)\left(1 - e^{-1}(1 - p_1)\right) & \\ \quad + e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\left(1 - e^{-1} - \frac{1}{2}(1 - p_1)e^{-1}\right) & t \geq \frac{1}{\lambda}, \end{cases} \tag{35}$$

Proof. Symmetric to Lemma 13. □

Given Lemmas 13 and 14, we have the following lemma:

Lemma 15. *There exists $\hat{t}_W := \frac{1}{\lambda}\left(1 - \ln\frac{2}{3}\right)$ such that, given the current policy set profile (θ_S, θ_W) such that $\theta_S = \theta_W = (\bar{X}, \bar{X})$, candidate W enters at $(1, \bar{X})$ or $(\bar{X}, \bar{X})$ for each $-t \in (-\hat{t}_W, 0]$ and he enters at $(\bar{X}, 1)$ or $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -\hat{t}_W)$. Moreover, candidate S's value of candidate W entering at $(1, \bar{X})$ is given by (34) and her value of him entering at $(\bar{X}, 1)$ is given by (35).*

Candidate S copies candidate W's policy as soon as possible after he enters at $(1, \bar{X})$ or $(\bar{X}, 1)$. The probability that S can copy W's policy position approaches 1 as the current time tends to $-\infty$. If S can copy W's policy before timing $-\frac{1}{\lambda}$, then W's payoff is $e^{-1}(1 - p_k)$, where k is the issue for which W does not specify his policy. Hence, far from the deadline, it is optimal to enter at $(\bar{X}, 1)$ (being ambiguous in the issue for which p_k is low) if he ever enters.

In contrast, if the deadline is close and S is unlikely to be able to copy, then entering at $(1, \bar{X})$ (being clear in the issue for which p_k is low) is optimal.

Proof. Given Lemmas 13 and 14, candidate W becomes indifferent between entering at $(1, \bar{X})$ and $(\bar{X}, 1)$ at $-\hat{t}_W$ where $\hat{t}_W$ is the unique solution for

$$
e^{-\lambda t}p_1 + \left(1 - e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\right)\left(1 - e^{-1}(1 - p_2)\right) + e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\left(1 - e^{-1} - \frac{1}{2}(1 - p_2)e^{-1}\right)
$$
$$
= e^{-\lambda t}p_2 + \left(1 - e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\right)\left(1 - e^{-1}(1 - p_1)\right) + e^{-\lambda\left(t - \frac{1}{\lambda}\right)}\left(1 - e^{-1} - \frac{1}{2}(1 - p_1)e^{-1}\right).
$$

Equivalently,

$$
\lambda\hat{t}_W = 1 - \ln\frac{2}{3} > 1.
$$

We now prove that entering at $(0, \bar{X})$ is strictly worse than $(1, \bar{X})$ for candidate W for each $-t \in (-\infty, 0]$. To this end, we calculate an upper bound of W's payoff when he enters at $(0, \bar{X})$. Since the game is constant-sum, an upper bound can be calculated by assuming that (i) S will enter at $(0, \bar{X})$ as soon as possible when W's current policy is $(0, \bar{X})$ and (ii) S will follow the equilibrium strategy of the one-issue case once $(\theta_S, \theta_W) = \left((0, \bar{X}), (0, \bar{X})\right)$ is realized. By Lemma 11, W does not enter at $(0, 0)$ or $(0, 1)$ after W enters at $(0, \bar{X})$, as long as S stays at $(\bar{X}, \bar{X})$. Moreover, once $(\theta_S, \theta_W) = \left((0, \bar{X}), (0, \bar{X})\right)$ is realized, W's best response against S's continuation strategy described in (ii) above is to follow the equilibrium strategy of the one-issue case. Hence,

the upper bound is characterized by the following dynamics: (i) S will enter at $(0, \bar{X})$ as soon as possible, (ii) W does not enter as long as S is at $(\bar{X}, \bar{X})$, and (iii) both candidates will follow the equilibrium strategy of the one-issue case once $(\theta_S, \theta_W) = ((0, \bar{X}), (0, \bar{X}))$ is realized.

In contrast, given Lemma 13, W's payoff of entering at $(1, \bar{X})$ is characterized by the following dynamics: (i) S will enter at $(1, \bar{X})$ as soon as possible, (ii) W does not enter as long as S is at $(\bar{X}, \bar{X})$, and (iii) both candidates will follow the equilibrium strategy of the one-issue case once $(\theta_S, \theta_W) = ((1, \bar{X}), (1, \bar{X}))$ is realized.

Note that once S has an opportunity to enter after W's entry, W's continuation payoff does not depend on whether he enters at $(0, \bar{X})$ or $(1, \bar{X})$. This implies that the difference of the continuation payoff given W's entering at $(0, \bar{X})$ and his entering at $(1, \bar{X})$ is solely due to the event in which S does not have an opportunity after W's entry. Under such an event, however, W's payoff is strictly greater under $(1, \bar{X})$ than under $(0, \bar{X})$. Therefore, entering at $(0, \bar{X})$ is strictly worse than $(1, \bar{X})$ for candidate W for each $-t \in (-\infty, 0]$.

Moreover, an argument analogous to the above (where we use Lemma 14 instead of Lemma 13) implies that entering at $(\bar{X}, 0)$ is strictly worse than $(\bar{X}, 1)$ for candidate W for each $-t \in (-\infty, 0]$. $\qquad\square$

M.3.2 Incentive to Enter

Define
$$t_W^* = \frac{1}{\lambda} \frac{5 - 6p_1 - p_2}{3(1 - 2p_1 + p_2)} \in \left(\frac{1}{\lambda}, \hat{t}_W\right).$$

Let $\bar{\sigma}$ be a strategy profile such that candidate W stays at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -t_W^*)$ and enters at $(1, \bar{X})$ for each $-t \in (-t_W^*, 0]$ and candidate S stays at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, 0]$ (and both candidates follow the equilibrium strategy specified in Lemmas 12–15 once some candidate enters).

We first show that candidate W prefers not to enter if and only if the deadline is sufficiently far:

Lemma 16. *Candidate W strictly prefers to enter at $(1, \bar{X})$ for each $-t \in (-t_W^*, 0]$ and strictly prefers to stay at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -t_W^*)$ given that both candidates will follow $\bar{\sigma}$ in the continuation play and given $\theta_{S,t} = (\bar{X}, \bar{X})$ and $\theta_{W,t} = (\bar{X}, \bar{X})$.*

Proof. Given strategy profile $\bar{\sigma}$, let $\hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right)$ be candidate W's value:

$$
\hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right) = \begin{cases} \int_0^t \lambda e^{-\lambda x} v_{W,t-x}\left((\bar{X},\bar{X}),(1,\bar{X})\right) dx & \text{for } t \leq t_W^*, \\ v_{W,t_W^*}\left((\bar{X},\bar{X}),(1,\bar{X})\right) & \text{for } t \geq t_W^*. \end{cases}
$$

In contrast, by (34) and the constant-sum assumption, we have

$$
v_{W,t}\left((\bar{X},\bar{X}),(1,\bar{X})\right) = \begin{cases} e^{-\lambda t}(1-p_1) + \frac{1}{2}t^2\lambda^2 e^{-\lambda t}(1-p_2) & t \leq \frac{1}{\lambda}, \\ \left(1 - e^{-\lambda\left(t-\frac{1}{\lambda}\right)}\right)e^{-1}(1-p_2) & \\ +e^{-\lambda\left(t-\frac{1}{\lambda}\right)}\left(e^{-1}(1-p_1) + \frac{1}{2}e^{-1}(1-p_2)\right) & t \geq \frac{1}{\lambda}. \end{cases}
$$

By algebra, we can show that the smallest solution to

$$
v_{W,t}\left((\bar{X},\bar{X}),(1,\bar{X})\right) - \hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right) = 0
$$

is in fact t_W^*. Hence, candidate W strictly prefers to enter at $(1,\bar{X})$ for each $-t \in (-t_W^*, 0]$.

The values $v_{W,t}\left((\bar{X},\bar{X}),(1,\bar{X})\right)$ and $v_{W,t}\left((\bar{X},\bar{X}),(\bar{X},1)\right)$ are decreasing in t. Hence, W strictly prefers not to enter for each $-t \in (-\infty, -t_W^*)$. $\qquad\square$

We next show that candidate S never enters given $\bar{\sigma}_W$:

Lemma 17. *For each $-t \in (-\infty, 0]$, given that both candidates follow the continuation strategy profile $\bar{\sigma}$ for each $-\tau \in (-t, 0]$, candidate S strictly prefers to stay at $(\bar{X},\bar{X})$ given $\theta_{S,t} = (\bar{X},\bar{X})$ and $\theta_{W,t} = (\bar{X},\bar{X})$.*

Proof. Define $\hat{v}_{S,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right)$ to be S's value at time $-t$ given the strategy profile $\bar{\sigma}$.

For $-t \in [-\frac{1}{\lambda}, 0]$, given the continuation strategy $\bar{\sigma}$, staying at $(\bar{X},\bar{X})$ is a best response for candidate S if and only if

$$
\underbrace{1 - \hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right)}_{=\hat{v}_{S,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right) \text{ by constant-sum}} \geq v_{S,t}\left((1,\bar{X}),(\bar{X},\bar{X})\right).
$$

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By algebra, this is equivalent to

$$
\begin{cases}
e^{-\lambda t}\left(p_1 - (p_2 - p_1)\lambda t - (1 - p_2)\frac{1}{6}\lambda^3 t^3\right) \geq 0 & \text{for } \lambda t \leq \frac{1-p_1-p_2}{1-p_2} \\
(1 - p_2)e^{-\frac{1-p_1-p_2}{1-p_2}} + e^{-\lambda t}\left(-(1 - p_1)\lambda t - (1 - p_2)\frac{1}{6}\lambda^3 t^3\right) \geq 0 & \text{for } 1 \geq \lambda t \geq \frac{1-p_1-p_2}{1-p_2}
\end{cases}
$$

Since

$$
\min_{0<p_1<p_2<\frac{1}{2}, \lambda t \leq \frac{1-p_1-p_2}{1-p_2}} p_1 - (p_2 - p_1)\lambda t - (1 - p_2)\frac{1}{6}\lambda^3 t^3 = \frac{7 + 27e + 21e^2 - 7e^3}{6\left(2 + 6e + 6e^2 + 2e^3\right)} \geq 0.154,
$$

and

$$
\min_{0<p_2<\frac{1}{2}, \frac{1-p_1-p_2}{1-p_2} \leq \lambda t \leq 1} (1 - p_2)e^{-\frac{1-p_1-p_2}{1-p_2}} + e^{-\lambda t}\left(-(1 - p_1)\lambda t - (1 - p_2)\frac{1}{6}\lambda^3 t^3\right)
$$

$$
= \frac{-\frac{13}{2} + 3e^{\frac{2}{1+e}} + \frac{6}{1+e}}{62} \geq 0.004,
$$

candidate S strictly prefers not to enter at each $-t \in [-\frac{1}{\lambda}, 0]$.

For $-t \in [-t_W^*, -\frac{1}{\lambda}]$, candidate S prefers $(\bar{X}, \bar{X})$ if

$$
\hat{v}_{S,t}\left((\bar{X}, \bar{X}), (\bar{X}, \bar{X})\right) \geq v_{S,t}\left((1, \bar{X}), (\bar{X}, \bar{X})\right).
$$

Define

$$
f(t, p_1, p_2) = \hat{v}_{S,t}\left((\bar{X}, \bar{X}), (\bar{X}, \bar{X})\right) - v_{S,t}\left((1, \bar{X}), (\bar{X}, \bar{X})\right).
$$

By algebra, we can show that

$$
\begin{aligned}
\frac{\partial^2 f}{\partial t^2}(t, p_1, p_2) &= 3 + 3p_1 - 6p_2 \geq 0, \\
\lim_{t\to\infty} \frac{\partial f}{\partial t}(t, p_1, p_2) &= 0, \\
\lim_{t\to\infty} f(t, p_1, p_2) &= \frac{-1 + p_1 + e^{\frac{p_1}{1-p_2}}(1 - p_2)}{e}.
\end{aligned}
$$

The first two lines imply $\frac{\partial f}{\partial t}(t, p_1, p_2) \leq 0$. Hence, we have

$$
\begin{aligned}
f(t, p_1, p_2) &\geq \min_{\frac{1}{1+e} \leq p_1 \leq p_2 \leq \frac{1}{2}} \lim_{t \to \infty} f(t, p_1, p_2) = \min_{\frac{1}{1+e} \leq p_1 \leq p_2 \leq \frac{1}{2}} \frac{-1 + p_1 + e^{\frac{p_1}{1-p_2}}(1 - p_2)}{e} \\
&= \frac{-1 + \frac{1}{1+e} + \frac{1}{2}e^{2\frac{e}{1+e}}}{e} = 0.047.
\end{aligned}
$$

Hence, S strictly prefers to stay at $(\bar{X}, \bar{X})$ for each $-t \in [-t_W^*, -\frac{1}{\lambda}]$.

For $-t < -t_W^*$, if candidate S enters, she obtains the payoff of $v_{S,t}\left((1, \bar{X}), (\bar{X}, \bar{X})\right) = v_{S,t_W^*}\left((1, \bar{X}), (\bar{X}, \bar{X})\right)$ given $t_W^* > \frac{1}{\lambda}$ and Lemma 12. In contrast, not entering for all times in $[-t, -t_W^*]$ gives her the payoff of $v_{S,t_W^*}\left((\bar{X}, \bar{X}), (\bar{X}, \bar{X})\right)$. Hence, given W's strict incentive to stay at $(\bar{X}, \bar{X})$ at time $-t_W^*$, S strictly prefers to stay at $(\bar{X}, \bar{X})$ for each $-t \in (-\infty, -t_W^*]$.

In total, we have shown that S prefers to stay at $(\bar{X}, \bar{X})$, as desired. $\qquad\square$

Note that the unique optimal strategy given that candidates will take $\bar{\sigma}$ in the continuation play is in fact $\bar{\sigma}$ by Lemmas 16 and 17. Hence, by continuous-time backward induction, in any PBE must have properties required for $\bar{\sigma}$.

M.4 Comparative Statics

By algebra, we have

$$
\begin{aligned}
\frac{\partial t_W^*}{\partial p_1} &= \frac{\partial}{\partial p_1}\left(\frac{1}{\lambda}\frac{5 - 6p_1 - p_2}{3(1 - 2p_1 + p_2)}\right) = \frac{4}{3\lambda}\frac{1 - 2p_2}{(p_2 - 2p_1 + 1)^2} > 0, \\
\frac{\partial t_W^*}{\partial p_2} &= \frac{\partial}{\partial p_2}\left(\frac{1}{\lambda}\frac{5 - 6p_1 - p_2}{3(1 - 2p_1 + p_2)}\right) = \frac{2}{3\lambda}\frac{4p_1 - 3}{(p_2 - 2p_1 + 1)^2} < 0.
\end{aligned}
$$

Let us provide the intuition for those results.

To see the effect of increasing p_1, take the partial derivative of W's value of entering and his value of not entering with respect to p_1. If he enters, he obtains the payoff that depends on p_1 (namely, $1 - p_1$) only if S cannot move. Hence, we have

$$
\frac{\partial}{\partial p_1} v_{W,t_W^*}\left((\bar{X}, \bar{X}), (1, \bar{X})\right) = -e^{-\lambda t_W^*}.
$$

If he does not enter, he obtains the payoff that depends on p_1 (again, $1 - p_1$) only if W enters and

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S cannot enter afterwards:

$$\frac{\partial}{\partial p_1}\hat{v}_{W,t_W^*}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right) = -\int_0^{t_W^*}\lambda e^{-\lambda x}e^{-\lambda\left(t_W^*-x\right)}dx = -\lambda t e^{-\lambda t_W^*} < -e^{-\lambda t_W^*} \text{ because } \lambda t_W^* > 1.$$

Hence, compared to the payoff of entering, the payoff of not entering decreases more in p_1. Since t_W^* is the smallest of the t's such that $v_{W,t}\left((\bar{X},\bar{X}),(1,\bar{X})\right) = \hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right)$, this implies that candidate W is willing to enter earlier as p_1 becomes larger (that is, $\partial t_W^*/\partial p_1 > 0$).

Similarly, we take the partial derivative of W's value of entering and his value of not entering with respect to p_2. If he enters, he obtains the payoff that depends on p_2 (namely, $1 - p_2$) only if S enters, then W enters, and then S cannot move after that. Hence, we have

$$\frac{\partial}{\partial p_2}v_{W,t_W^*}\left((\bar{X},\bar{X}),(1,\bar{X})\right) = -\int_0^{t_W^*}\lambda e^{-\lambda x}\int_0^{t_W^*-x}e^{-\lambda y}e^{-\lambda\left(t_W^*-x-y\right)}dy\,dx = -\lambda^2(t_W^*)^2 e^{-\lambda t_W^*}.$$

If he does not enter, he obtains the payoff that depends on p_2 (again, $1 - p_2$) only if W enters, then S enters, then W enters, and then S cannot enter after that:

$$\frac{\partial}{\partial p_2}\hat{v}_{W,t_W^*}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right) = -\int_0^{t_W^*}\lambda e^{-\lambda x}\int_0^{t_W^*-x}e^{-\lambda y}\int_0^{t_W^*-x-y}e^{-\lambda z}e^{-\lambda\left(t_W^*-x-y-z\right)}dy\,dx = -\frac{1}{6}\lambda^3(t_W^*)^3 e^{-\lambda t_W^*}.$$

Since $\frac{1}{6}\lambda^3(t_W^*)^3 < \lambda^2(t_W^*)^2$, this implies that, compared to the payoff of entering, the payoff of not entering decreases less in p_2. Hence, since t_W^* is the smallest of the t's such that $v_{W,t}\left((\bar{X},\bar{X}),(1,\bar{X})\right) = \hat{v}_{W,t}\left((\bar{X},\bar{X}),(\bar{X},\bar{X})\right)$, candidate W postpones his entering time as p_2 becomes larger (that is, $\partial t_W^*/\partial p_2 < 0$).

N   Analysis for Remark 11

N.1   When S is an Incumbent with a Pre-Specified Policy

Suppose that S is an incumbent, who has already specified her policy for the first issue during her term in the office. Suppose she has picked $x_1 = 1$. In contrast, W is a challenger who has not specified any policy. In this game, the following result holds:

Proposition 12. *Suppose $p_k \geq \frac{1}{1+e}$ for each $k \in \{1,2\}$. In any PBE, the following hold:*

1. Given $\left((1,\bar{X}),(\bar{X},\bar{X})\right)$, (i) candidate S stays at $(1,\bar{X})$ for each $-t \in (-\infty,0]$ and (ii)

candidate W stays at $\left(\bar{X}, \bar{X}\right)$ for each $-t \in (-\infty, -\frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2})$ and enters at $(1,1)$ for each $-t \in (-\frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2}, 0]$.

2. *Given $\left((1, \bar{X}), (1,1)\right)$, (i) candidate S enters at $(1,1)$ as soon as possible.*

Moreover, the ex ante payoff of candidate S when the game starts at time $-t$ is given by

$$
\begin{cases}
1 - e^{-\lambda t}\left(p_1 + (1-p_2)\lambda t\right) & \text{if } t \leq \frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2}, \\
1 - (1-p_2)e^{-1+\frac{p_1}{1-p_2}} & \text{if } t \geq \frac{1}{\lambda}\frac{1-p_1-p_2}{1-p_2}.
\end{cases}
$$

Proof. Since Lemma 12 shows there is a unique Markov equilibrium (except for measure-zero cutoff times) in the subgame after $(1, \bar{X}), (\bar{X}, \bar{X})$, this proposition is a direct corollary of Lemma 12. □

For issue 2 (where S has not specified her policy), as in the case with the one-issue case, candidate W waits until a cutoff time and then enters at $\{1\}$; and S waits until W enters. However, for issue 1, when W enters, his policy is matched with candidate S's. The reason is as follows: The most likely event is that the voter will be located at $(\{1\}, \{1\})$. In this case, if candidate W does not enter or enters at $\{0\}$ in issue 1 (and he enters at $\{1\}$ for issue 2), then the voter's utility from W is at most

$$
\max\left\{\frac{1}{2} \times (-1) + \frac{1}{2} \times 0, -1\right\} = -\frac{1}{2},
$$

while the voter's utility from S (given that S does not move after W enters) is

$$
\frac{1}{2} \times (-1) + \frac{1}{2} \times 0 + \delta = -\frac{1}{2} + \delta.
$$

Hence, W cannot win. In contrast, if W enters at $(\{1\}, \{1\})$, the voter's utility from W is 0 and W can win.

Intuitively, if S cannot move after W enters, entering at $\{1\}$ for issue 2 gives W an advantage to overcome the valence (in a likely event that the voter prefers policy $\{1\}$ for issue 2) if W enters at $\{1\}$ for issue 1 and only difference between S and W is the policy for issue 2. In contrast, if W does not enter or enters at $\{0\}$ for issue 1, in a likely event that the voter prefers the policy $\{1\}$ for issue 1, it gives advantage to S. To avoid this, W in fact should enter at $\{1\}$ for both issues.

N.2 Cost of the Commitment

In order to calculate the cost of the commitment, let us first calculate S's payoff from the policy announcement timing game: Recall that, in any PBE, W enters at $(1, \bar{X})$ for $(-t_W^*, 0]$, and S copies it as soon as possible. Once S copies, then the analysis is the same as the one-issue case with $p = p_2$.

In the one-issue case, the equilibrium payoff for S at time $-t$ is

$$
\begin{aligned}
v_t &= f(t) \equiv \int_{x=0}^{t} \lambda e^{-\lambda x} \left(e^{-\lambda(t-x)} p_2 + \left(1 - e^{-\lambda(t-x)}\right) \right) dx + e^{-\lambda t} = 1 - (1 - p_2)\,\lambda t e^{-\lambda t} \text{ if } t < \frac{1}{\lambda}, \\
v_t &= f\left(\frac{1}{\lambda}\right) = 1 - (1 - p_2)\, e^{-1} \text{ if } t \geq \frac{1}{\lambda}.
\end{aligned}
$$

Given this value, the equilibrium payoff for S at time $-t \leq t_W^*$ in the one-issue case is

$$
\int_{x=0}^{t_W^* - \frac{1}{\lambda}} \lambda e^{-\lambda x} \left(\int_{y=0}^{t_W^* - x - \frac{1}{\lambda}} \lambda e^{-\lambda y} f\left(\frac{1}{\lambda}\right) + e^{-\lambda\left(t_W^* - x - \frac{1}{\lambda}\right)} \int_{y=0}^{\frac{1}{\lambda}} \lambda e^{-\lambda y} f\left(\frac{1}{\lambda} - y\right) dy + e^{-\lambda\left(t_W^* - x\right)} p_1 \right) dx
$$

$$
+ e^{-\lambda\left(t_W^* - \frac{1}{\lambda}\right)} \int_{x=0}^{\frac{1}{\lambda}} \lambda e^{-\lambda x} \left(\int_{y=0}^{\frac{1}{\lambda} - x} \lambda e^{-\lambda y} f\left(\frac{1}{\lambda} - x - y\right) + e^{-\lambda\left(\frac{1}{\lambda} - x\right)} p_1 \right) dx + e^{-\lambda t_W^*}
$$

$$
= \frac{e^{\frac{6p_1 + p_2 - 5}{-6p_1 + 3p_2 + 3}} \left(-8 p_1 p_2 + 12(p_1 - 1)p_1 - 3\,(p_2)^2 + 10 p_2 + 1 \right)}{12 p_1 - 6(p_2 + 1)} + \frac{p_2 - 1}{e} + 1
$$

if $t_W^* \leq t$.

Thus, fo any sufficiently large t, the difference is

$$
\begin{aligned}
f(p_1, p_2) &\equiv \frac{e^{\frac{6p_1 + p_2 - 5}{-6p_1 + 3p_2 + 3}} \left(-8 p_1 p_2 + 12(p_1 - 1)p_1 - 3\,(p_2)^2 + 10 p_2 + 1 \right)}{12 p_1 - 6(p_2 + 1)} + \frac{p_2 - 1}{e} + 1 \\
&\quad - \left(1 - (1 - p_2)\, e^{-1 + \frac{p_1}{1 - p_2}} \right).
\end{aligned}
$$

By algebra, we can show that, for $p_1 \geq \frac{1}{1+e}$, $p_2 \geq \frac{1}{1+e}$, and $p_1 \leq p_2$, we have $\partial f(p_1, p_2)/\partial p_1 > 0$ and $\partial f(p_1, p_2)/\partial p_2 < 0$. The intuition for those comparative statics are explained in Remark 11.

O   Proofs and Additional Discussions for Appendix G

O.1   Proof of Proposition 11

First, we compute a lower bound of the probability of candidate i winning conditional on her being able to move at time $-t$. To calculate such a bound, suppose candidate i does not enter for each time in the time interval $(-t, -\tau)$, and then enters for each time in the time interval $[-\tau, 0]$. A lower bound of the probability of winning when i uses this strategy, denoted by $\bar{p}_\tau$, is given by the following consideration: Since the second entrant can win for sure, i's minimum winning probability is given by the assumption that her opponent j will not enter until i enters. The bound can be computed as follows:

$$\bar{p}_\tau = \int_0^\tau \underbrace{\lambda_i e^{-\lambda_i s}}_{i \text{ enters at } -(\tau-s)} \times \underbrace{e^{-\lambda_j(\tau-s)}}_{j \text{ cannot enter after } i \text{ enters}} ds = \begin{cases} \dfrac{\lambda_i \left[e^{-\lambda_i \tau} - e^{-\lambda_j \tau} \right]}{\lambda_j - \lambda_i} > 0 & \text{if } \lambda_i \neq \lambda_i \\[4mm] \lambda_i \tau e^{-\lambda_i \tau} & \text{if } \lambda_i = \lambda_j \end{cases}.$$

Another lower bound can be calculated by assuming that i enters at time $-t$, and it is given by $e^{-\lambda_j t}$. Hence, in total, we obtain a bound of $\max\{e^{-\lambda_j t}, \max_{\tau \in [0,t]} \bar{p}_\tau\}$. This implies that, if we take $\varepsilon < \frac{\min_{t \in [0,\infty)} \max\{e^{-\lambda_j t}, \max_{\tau \in [0,t]} \bar{p}_\tau\}}{\max_{x,y \in X} |u_i(x) - u_i(y)|}$, then at every time $-t$, there exists a strictly better strategy for candidate i than entering at a policy with which i will lose for sure.

These bounds can be used to derive an explicit expression of $\bar{\varepsilon}$:

$$\bar{\varepsilon} = \min \left\{ 1, \min_{i \in \{L,R\}} \frac{\min_{t \in [0,\infty)} \max\{e^{-\lambda_j t}, \max_{\tau \in [0,t]} \bar{p}_\tau\}}{\max_{x,y \in X} |u_i(x) - u_i(y)|} \right\}. \tag{36}$$

Given this definition of $\bar{\varepsilon}$, $\varepsilon < \bar{\varepsilon}$ ensures that it is a dominated strategy for candidate i to enter at a policy x such that i loses at a policy set profile $(\{x\}, X)$.

We next derive the set of policies with which candidate i can win given that candidate j has entered at x, which we denote by $X(i, x)$. If candidate j's policy is $x \in X$, candidate i can win if and only if her policy is x' (including the case where she picks X and her ideal policy is x') satisfying one of the following three conditions:

1. $x_1 \leq x'_1$ and $x_2 \leq x'_2$ (voters at $(1,0)$ and $(0,1)$ vote for her);

2. $x_1 \leq x'_1$ and $x'_1 + x'_2 \leq x_1 + x_2$ (voters at $(1,0)$ and $(0,0)$ vote for her); or

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3. $x_2 \leq x_2'$ and $x_1' + x_2' \leq x_1 + x_2$ (voters at $(0,1)$ and $(0,0)$ vote for her).

Second, we derive the set of policies with which candidate L can win if candidate R does not enter. Since the voters believe that candidate R implements $\left(\frac{1}{2}, \frac{1}{2}\right)$ if she does not enter, the set is the same as $X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$. Similarly, candidate R can win with policies in $X\left(R, (0,0)\right)$ if candidate L does not enter.

We now consider each candidate's best response to the opponent's entry to x. First, suppose that R has entered at x. Candidate L enters at (x_1, x_2') with $x_2' \leq x_1$ if $x_1 \leq x_2$, and (x_1', x_2) with $x_1' \leq x_2$ if $x_1 \geq x_2$. Given the tie breaking rule, we conclude that candidate L enters at $(\min\{x_1, x_2\}, \min\{x_1, x_2\})$.

Second, suppose that L has entered at x. Given this, suppose that R's entry to x' is a best response.

1. If $x_1 \leq x_1'$ and $x_2 \leq x_2'$, then the following hold.

   (a) If $x_1 \leq \frac{1}{2}$ and $x_2 \leq \frac{1}{2}$, then $x' = \left(\frac{1}{2}, \frac{1}{2}\right)$. In this case, she receives $u_R(x') = \frac{1}{2}$.

   (b) Otherwise, given the tie breaking rule, x' is on the line segment connecting $(0,1)$ and $(1,0)$. In particular, $x_1' = x_1$ and $x_2' = 1 - x_1$ if $x_1 > \frac{1}{2}$; and $x_1' = 1 - x_2$ and $x_2' = x_2$ if $x_2 > \frac{1}{2}$. In this case, she receives $u_R(x') = 1 - \max\{x_1, x_2\}$.

2. If $x_1' + x_2' \leq x_1 + x_2$, then $x' = \left(\frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2}\right)$ and she receives $u_R(x') = \frac{x_1 + x_2}{2}$.

Hence, for $x \in X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$ (L never enters outside of $X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$), R enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$ if $x = \left(\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right)$; she enters at x' with $x_1 \leq x_1'$ and $x_2 \leq x_2'$ if x satisfies $x_1 \leq \frac{1}{2}$ and $x_2 \leq \frac{1}{2}$ or x satisfies

$$\frac{x_1 + x_2}{2} \leq 1 - \max\{x_1, x_2\}; \tag{37}$$

and she enters at $\left(\frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2}\right)$ if x satisfies $\frac{x_1 + x_2}{2} \geq 1 - \max\{x_1, x_2\}$ and $x \neq \left(\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right)$. Since $x_1 + x_2 = \max\{x_1, x_2\} + \min\{x_1, x_2\}$, (37) is equivalent to

$$\min\{x_1, x_2\} \leq 2 - 3\max\{x_1, x_2\}.$$

Given this response of the other candidate, the following property holds for L. To formalize, let $X_t^L \subseteq X$ be the set of policies such that $x \in X_t^L$ if and only if L's continuation payoff is maximized

if he enters at x at time $-t$ conditional on the event that R has not entered and L enters at $-t$.

Let t_L^1 be the solution for

$$e^{-\lambda_R t} = \frac{1}{2}. \tag{38}$$

Lemma 18. $X_t^L = \left\{\left(\frac{1}{2}, \frac{1}{2}\right)\right\}$ for $-t \in (-t_L^1, 0]$, and $X_t^L = \left\{\left(\frac{2}{3}, 0\right), \left(0, \frac{2}{3}\right)\right\}$ for $-t \in \left(-\infty, -t_L^1\right)$.

Proof. First, note that candidate L does not enter at any policy $x \notin X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$ since R's best response against such x guarantees L to get a payoff of $\varepsilon u_L(\frac{1}{2}, \frac{1}{2})$, which is dominated by the payoff from a strategy of entering at $\left(\frac{1}{2}, \frac{1}{2}\right) \in X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$. Second, $X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right) = \{x \in X \mid \max\{x_1, x_2\} \geq \frac{1}{2}\}$ holds. Third, we consider the following three exhaustive cases, where cases are defined depending on which policy among $X\left(L, \left(\frac{1}{2}, \frac{1}{2}\right)\right)$ candidate L enters at:

1. If L enters at $\left(\frac{1}{2}, 0\right)$ or $\left(0, \frac{1}{2}\right)$, then R will win and implement $\left(\frac{1}{2}, \frac{1}{2}\right)$ if she enters afterward. Hence, L's payoff is

$$\underbrace{e^{-\lambda_R t}}_{\substack{\text{Probability of } R \text{ not} \\ \text{receiving an opportunity}}} + \varepsilon \underbrace{\left(-\frac{1}{2}\right)}_{\text{Utility from the policy is } -\frac{1}{2} \text{ anyway}}.$$

2. If L enters at x with $\min\{x_1, x_2\} \leq 2 - 3\max\{x_1, x_2\}$, then R, if she enters afterward, will win and implement (x_1', x_2') such that $x_1' = x_1$ and $x_2' = 1 - x_1$ if $x_1 > \frac{1}{2}$, and $x_1' = 1 - x_2$ and $x_2' = x_2$ if $x_2 > \frac{1}{2}$. Hence, L's payoff is

$$\underbrace{e^{-\lambda_R t}}_{\substack{\text{Probability of } R \text{ not} \\ \text{receiving an opportunity}}} + \varepsilon \underbrace{\left(-\max\{x_1, x_2\}\right)}_{\substack{\text{Utility from the policy is } -\max\{x_1,x_2\} \text{ anyway} \\ \text{since } \max\{x_1,x_2\}=\max\{x_1',x_2'\}}}.$$

Thus, among all x's in this case, L's payoff is maximized if and only if he enters at $\left(\frac{1}{2}, 0\right)$, $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, or any convex combination of them, and his payoff is then

$$e^{-\lambda_R t} + \varepsilon\left(-\frac{1}{2}\right).$$

3. If L enters at x with $\min\{x_1, x_2\} \geq 2 - 3\max\{x_1, x_2\}$, then R will win and implement

$\left(\frac{x_1+x_2}{2}, \frac{x_1+x_2}{2}\right)$. Hence, L's payoff is

$$e^{-\lambda_R t} - \varepsilon e^{-\lambda_R t} \left(\max\{x_1, x_2\}\right) + \varepsilon \left(1 - e^{-\lambda_R t}\right) \left(-\frac{x_1 + x_2}{2}\right).$$

If (x_1, x_2) is the optimal policy for L under this case, then the constraint $\min\{x_1, x_2\} \geq 2 - 3 \max\{x_1, x_2\}$ has to bind, since otherwise L wants to reduce $\max\{x_1, x_2\}$. The set of x's satisfying $\min\{x_1, x_2\} = 2 - 3 \max\{x_1, x_2\}$ is expressed as

$$\left\{ \left(\frac{2}{3} - \theta, 3\theta\right) \cup \left(3\theta, \frac{2}{3} - \theta\right) : \text{there exists } \theta \geq 0 \text{ and } \frac{2}{3} - \theta \geq 3\theta \right\}.$$

Given θ, L's payoff is equal to

$$e^{-\lambda_R t} - \varepsilon e^{-\lambda_R t} \left(\frac{2}{3} - \theta\right) - \varepsilon \left(1 - e^{-\lambda_R t}\right) \left(\frac{\frac{2}{3} - \theta + 3\theta}{2}\right)$$

$$= e^{-\lambda_R t} - \varepsilon e^{-\lambda_R t} \left(\frac{2}{3} - \theta\right) - \varepsilon \left(1 - e^{-\lambda_R t}\right) \left(\frac{1}{3} + \theta\right).$$

Hence, if $e^{-\lambda_R t} \geq \frac{1}{2}$, then it is the best for L to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$; and if $e^{-\lambda_R t} \leq \frac{1}{2}$, then it is the best for him to enter at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$.

In total, for $-t \in (-t_L^1, 0]$, candidate L enters at $\left(\frac{1}{2}, 0\right)$, $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, or any convex combination of them, and obtains a payoff of $e^{-\lambda_R t} - \varepsilon \frac{1}{2}$. Again, by the tie breaking rule, L enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

$\square$

In addition, the following property holds for R:

Lemma 19. *For all $-t \in (-\infty, 0]$, if L's strategy is such that he enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$ for all times in $(-t, 0]$, then under any best response by R to such a strategy, R does not enter at $-t$.*

Proof. Fix $-t$. Let σ_R^* be a strategy of R such that, conditional on there being no entry by any candidate no later than time $-t$, R does not enter unless L enters, and best-responds to L's policy once L enters. Consider the following two cases:

1. Conditional on the event under which L will have an opportunity at some $-\tau \in (-t, 0]$, (i) if R enters at $-t$, her payoff will be at most $\varepsilon\frac{1}{2}$, but (ii) σ_R^* gives her a payoff strictly greater than $\varepsilon\frac{1}{2}$ (since L enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and R can win if she can enter after L enters).

2. Conditional on the event under which L will not enter, at $-t$, both entering at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and following σ_R^* are optimal for R while entering at any other policy is strictly worse.

Since the first event happens with strictly positive probability, the proof is complete. □

We now pin down the candidates' strategies at $-t$ sufficiently close to 0. Let t_L^2 be the unique t satisfying the following.

$$
\begin{cases}
\frac{\lambda_R e^{-\lambda_R t} - \lambda_L e^{-\lambda_L t}}{\lambda_L - \lambda_R} = 0 & \text{if } \lambda_L \neq \lambda_R, \\
t = \frac{1}{\lambda} & \text{if } \lambda_L = \lambda_R = \lambda.
\end{cases}
\tag{39}
$$

For each $t \in [0, \min\{t_L^1, t_L^2\})$, suppose that candidates take the following continuation play for each $-\tau \in (-t, 0]$: R does not enter unless L enters (and takes a static best-response once L enters) and L enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$. Then, we show that, at time $-t$, it is optimal for R not to enter at $-t$ and for L to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Given this continuation play, Lemma 19 ensures that R has a strict incentive not to enter at $-t$. Hence, we consider L's incentive. L's payoff when he does not enter at time $-t$ is

$$
\int_0^t \lambda_L e^{-\lambda_L \tau} \left(e^{-\lambda_R(t-\tau)} - \varepsilon\frac{1}{2} \right) d\tau - e^{-\lambda_L t} \left(-\varepsilon\frac{1}{2} \right) =
\begin{cases}
\lambda_L \frac{e^{-\lambda_R t} - e^{-\lambda_L t}}{\lambda_L - \lambda_R} - \varepsilon\frac{1}{2} & \text{if } \lambda_L \neq \lambda_R \\
e^{-\lambda t} \lambda t - \varepsilon\frac{1}{2} & \text{if } \lambda_L = \lambda_R = \lambda
\end{cases}.
$$

Hence, L strictly prefers to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$ at time $-t$ if the following holds: $t < t_L^1$ and

$$
\begin{cases}
\lambda_L \frac{e^{-\lambda_R t} - e^{-\lambda_L t}}{\lambda_L - \lambda_R} - \varepsilon\frac{1}{2} > e^{-\lambda_R t} - \varepsilon\frac{1}{2} & \text{if } \lambda_L \neq \lambda_R, \\
\lambda t > 1 & \text{if } \lambda_L = \lambda_R = \lambda
\end{cases}
\Leftrightarrow t < t_L^2.
$$

Moreover, if $t_L^2 \leq t_L^1$, then L is indifferent between entering and not entering at time $-t_L^2$.

Therefore, by the continuity of probabilities in time and boundedness of payoffs, the continuous-time backward induction implies that for each $t \in [0, \min\{t_L^1, t_L^2\})$, at time $-t$, it is uniquely optimal

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for R not to enter and for L to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$. In what follows, we consider candidates' incentives at time $-t$ with $t > \min\{t_L^1, t_L^2\}$.

If time $-t_L^2$ is after the time at which L's optimal entering policy switches from $\left(\frac{1}{2}, \frac{1}{2}\right)$ to $\left(0, \frac{2}{3}\right)$, that is, if $t_L^2 < t_L^1$, then neither L nor R enters for $-t < -t_L^2$. To see why, suppose this claim holds for $-\tau \in [-t, -t_L^2)$. Note that, on the one hand, L's payoff from entering at time $-t$ is strictly decreasing in t since the probability of candidate R entering afterward increases. On the other hand, given that R does not enter for each $-\tau$ with $\tau \leq t$, L can secure a payoff of

$$\begin{cases} \lambda_L \frac{e^{-\lambda_R t_L^2} - e^{-\lambda_L t_L^2}}{\lambda_L - \lambda_R} - \varepsilon \frac{1}{2} & \text{if } \lambda_L \neq \lambda_R \\ e^{-\lambda t_L^2} \lambda t_L^2 - \varepsilon \frac{1}{2} & \text{if } \lambda_L = \lambda_R = \lambda \end{cases}$$

by not entering in the time interval $[-t, -t_L^2)$. Since candidate L is indifferent between entering and not entering at $-t = -t_L^2$, he strictly prefers not entering for each $-t < -t_L^2$. With the same reasoning as Lemma 19, one can show that R strictly prefers not entering for each $-t < -t_L^2$. Hence, by the continuity of probabilities in time and boundedness of payoffs, the continuous-time backward induction implies that neither L nor R enters at any $-t < -t_L^2$ in any PBE.

Hence, we are left to consider the case in which $t_L^2 > t_L^1$. By the continuity of the continuation payoff in time, there exists $\varepsilon > 0$ such that candidate L enters at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ for each $-t \in \left(-t_L^1 - \varepsilon, -t_L^1\right)$. Given this behavior of candidate L, candidate R faces the following trade-off:

1. Conditional on the event under which L will enter after R, the only possibilities for best responses are entering at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and not entering. This is because entering at $x \neq \left(\frac{1}{2}, \frac{1}{2}\right)$ is not optimal, as after R's entry to $\left(\frac{1}{2}, \frac{1}{2}\right)$, L's unique best response is to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$, which is R's ideal policy. Since R always loses if L enters after R enters, if R enters, R should enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$. In particular, entering at $\left(\frac{2}{3}, 0\right)$ and entering at $\left(0, \frac{2}{3}\right)$ are both suboptimal. In contrast, if R does not enter, then L enters at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ for $(-t, -t_L^1)$.

2. Conditional on the event under which L will not enter, both entering at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and not entering are the best responses for R.

Note that the advantage for R to enter at $\left(\frac{1}{2}, \frac{1}{2}\right)$ is to change L's policy from $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ to $\left(\frac{1}{2}, \frac{1}{2}\right)$ (R's ideal policy). However, such an advantage is only valid when L enters after R enters.

Since L will win for sure in such a case, we will prove that, for sufficiently small policy preference $\varepsilon > 0$, it is uniquely optimal for R not to enter:

Lemma 20. *Suppose $t_L^1 < t_L^2$. Fix $-t < -t_L^1$ and L's strategy such that he enters at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ for all times in $\left(-t, -t_L^1\right)$, and enters at $\left(\frac{1}{2}, \frac{1}{2}\right)$ for all times in $\left(-t_L^1, 0\right]$. Then, conditional on any history at time $-t$ at which no candidate has entered and R receives an opportunity, not entering is R's unique best response.*

Proof. Fix time $-t < -t_L^1$. Since R not entering at all in the time interval $[-t, -t_L^1)$ is one of the feasible continuation strategies, it suffices to show that, for each $-t$, this strategy is strictly better for R than her entering at $-t$. Consider the following two cases:

1. L obtains an opportunity in the time interval $(-t, -t_L^1)$. Conditional on this event, if R enters at $x \in X$ at time $-t$, then L enters at $y(L, x)$ and wins for sure. Hence, assuming that R enters, the optimal policy for her to enter is $\left(\frac{1}{2}, \frac{1}{2}\right)$ and it gives R a payoff of $\varepsilon \frac{1}{2}$. Meanwhile, if R does not enter until $-t_L^1$, then R obtains

$$\underbrace{\left(1 - e^{-\lambda_R t_L^1}\right)}_{R \text{ can enter by the deadline after } t_L^1} \cdot \left(1 + \varepsilon \frac{1}{3}\right) + e^{-\lambda_R t_L^1} \varepsilon \cdot 0 \geq 1 - e^{-\lambda_R t_L^1}.$$

    Since (38) implies that $t_L^1 = \frac{\ln \frac{1}{2}}{-\lambda_R} = \frac{\ln 2}{\lambda_R}$ and (36) implies $\varepsilon < 1$, straightforward algebra shows that not entering is uniquely optimal for R at $-t$.

2. L does not obtain an opportunity in the time interval $(-t, -t_L^1)$. Conditional on this event, since R's unique best response is not to enter at time $-t_L^1$ by Lemma 19 (note that, conditional on the event that L does not obtain an opportunity in $(-t, -t_L^1)$, R wants to enter at $-t$ if and only if she wants to enter at $-t_L^1$), it is uniquely optimal for R not to enter at $-t$.

    Therefore, conditional on both events, it is uniquely optimal for R not to enter at time $-t$. $\quad\square$

Let $\bar{\sigma}_L$ be candidate L's strategy such that, if R has not entered, L enters at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ for each $-t \in \left(-\infty, -t_L^1\right)$ and at $\left(\frac{1}{2}, \frac{1}{2}\right)$ for each $-t \in \left(-t_L^1, 0\right]$ (and L chooses a static best response once R enters); and let $\bar{\sigma}_R$ be candidate R's strategy such that R never enters if L has not entered

(and R chooses a static best response once L enters). By $t_1^L < t_2^L$ and Lemma 20, there exists $\varepsilon > 0$ such that $\bar{\sigma}_i$ is optimal for each $-t \geq -t_L^1 - \varepsilon$ and $i \in \{L, R\}$.

For $t > t_L^1$, suppose that the candidates take $\bar{\sigma}$ for each time $-\tau$ with $\tau < t$. Given that R never enters after $-t$, given $t > t_L^1$, we must have $X_t^L = \{\left(\frac{2}{3}, 0\right), \left(0, \frac{2}{3}\right)\}$. Note that the probability that L wins by entering at $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$—equivalently, the probability that R cannot enter after L enters—is decreasing in t and converges to 0 as $t \to \infty$. Hence, the payoff of entering converges to $-\varepsilon\frac{1}{3}$. By (36), for sufficiently large t, there exists $\tau' \in [0, t]$ such that L can obtain a payoff greater than $-\varepsilon\frac{1}{3}$ by instead not entering until $-\tau'$. Hence, given the bound of ε we imposed and the continuity of the continuation payoff in time given $\bar{\sigma}$, there exists the smallest t such that L is indifferent between entering and not entering at $-t$. Let t_L^3 be such t.

By the continuity of probabilities in t and boundedness of the payoffs, the continuous-time backward induction implies that $\bar{\sigma}_i$ is optimal for any $-t > -t_L^3$ in any PBE. Hence, it remans for us to show that no candidate enters at $-t < -t_L^3$. Let σ^* be a pair of strategies such that neither L nor R enters at $-t < -t_L^3$ and both of them take $\bar{\sigma}$ for any $-t > -t_L^3$. One can show that R chooses a best response in the same way as in Lemma 20 given the continuation play σ^*. L's incentive can be checked as follows: Let v_L^3 be L's payoff of entering at time $-t_L^3$ given the continuation play $\bar{\sigma}$. Entering at $-t < -t_L^3$ gives him a payoff strictly lower than v_L^3 since the probability that R can enter after L enters increases monotonically in t. Not entering until $-t_L^3$ guarantees a payoff of v_L^3 since L is indifferent between entering and not entering at $-t_L^3$ given the continuation play σ^*. Hence, by the continuity of probabilities in time and boundedness of payoffs, the continuous-time backward induction implies that both candidates take σ^* in any PBE.

Finally, we examine the conditions under which we have $t_L^1 < t_L^2$ and $t_L^1 > t_L^2$, respectively. Note that (38) implies that $t_L^1 = \frac{\ln\frac{1}{2}}{-\lambda_R} = \frac{\ln 2}{\lambda_R}$. Since the left-hand side of (39) is negative for $t \in \left(0, t_L^2\right)$, and positive for $t > t_L^2$, we have $t_L^1 < t_L^2$ if and only if the left-hand side of (39) is negative for $t = t_L^1$. Substituting $t = t_L^1 = \frac{\ln 2}{\lambda_R}$, the left-hand side of (39) is equal to

$$\frac{\lambda_R e^{-\lambda_R\frac{\ln 2}{\lambda_R}} - \lambda_L e^{-\lambda_L\frac{\ln 2}{\lambda_R}}}{\lambda_L - \lambda_R} = \frac{\frac{1}{2} - \frac{\lambda_L}{\lambda_R}\left(\frac{1}{2}\right)^{\frac{\lambda_L}{\lambda_R}}}{\frac{\lambda_L}{\lambda_R} - 1}.$$

Letting $l = \frac{\lambda_L}{\lambda_R}$, this is equal to $\frac{\frac{1}{2} - l\left(\frac{1}{2}\right)^l}{l - 1}$. Taking the derivative of the numerator with respect to l

yields

$$-\left(\frac{1}{2}\right)^l + l\left(\frac{1}{2}\right)^l \ln 2 = \left(\frac{1}{2}\right)^l (1 - l \ln 2).$$

Hence, the numerator is decreasing for $l \leq \frac{1}{\ln 2}$ and increasing for $l \geq \frac{1}{\ln 2}$.

Note that the numerator is zero at $l = 1 < \frac{1}{\ln 2}$ and at $l = 2 > \frac{1}{\ln 2}$. Hence, $\frac{1}{2} - l\left(\frac{1}{2}\right)^l$ is positive for $l < 1$, 0 for $l = 1$, negative for $l \in (1, 2)$, 0 for $l = 2$, and positive for $l > 2$. Together with the denominator (and using l'Hopital rule at $l = 1$), we have

$$\frac{\frac{1}{2} - l\left(\frac{1}{2}\right)^l}{l - 1} \begin{cases} < 0 & \text{for } l \in (0, 2) \\ = 0 & \text{for } l = 2 \\ > 0 & \text{for } l > 2 \end{cases}.$$

Therefore, $t_L^1 < t_L^2$ if and only if $\frac{\lambda_L}{\lambda_R} < 2$. In a similar vein, one can show that $t_L^1 > t_L^2$ if and only if $\frac{\lambda_L}{\lambda_R} > 2$.

O.2 Persuasion-Cost Election Campaign

In the policy-motivated election campaign in Section G, L enters at suboptimal policies $\left(\frac{2}{3}, 0\right)$ or $\left(0, \frac{2}{3}\right)$ since, when R enters after L, this suboptimal policy will lead R to enter at a more favorable policy for L. Such a consideration does not occur if L does not care about what policy R picks when R wins. In such a case, the equilibrium dynamics are simpler than in the model in Section G, while we can still conduct comparative statics with respect to the distribution of voters and the ideal points of the candidates more easily, keeping the advantage of the policy-motivated model over the purely office-motivated model as in Section 2.2.

Let X be an arbitrary policy space that is a full-dimensional compact subset of $\mathbb{R}^n$ for some n, and recall that $|\cdot|$ denotes the Euclidian distance. A unit mass of voters are distributed over X according to the distribution $\mu(x)$ over X. The voter located at x has utility of $-|x - y|$ from policy y.

There are two candidates L and R. Given a profile of policies $(x_L, x_R) \in X \times X$, we define candidate i's vote share $S_i(x_L, x_R)$ and probability of i's winning $P_i(x_L, x_R)$ as in Section 2.2. The definition of $P_i(X_i, X_j)$ when $X_i = X$ or $X_j = X$ holds is given later. We assume that $(X, \mu) \notin \mathcal{M}$.

The ideal policies of candidates L and R are x_L^* and x_R^*, respectively. The ideal policies are

common knowledge among voters and candidates. The utility for candidate i is equal to

$$
\begin{cases}
\mathbb{I}_{i \text{ wins}} - \varepsilon \left| x_i^* - x \right| & \text{if } X_i = \{x\} \subseteq X \\
\mathbb{I}_{i \text{ wins}} & \text{if } X_i = X
\end{cases},
$$

where $\varepsilon > 0$. That is, each candidate incurs a cost $|x_i^* - x|$ associated with the policy to which she commits, regardless of whether she wins the election. For example, if the voters believe that x_i^* is i's ideal policy, committing to x far from x_i^* requires the cost of persuading the voters. Without specifying the policy—with $X_i = X$—, in contrast, she does not have to pay such a cost. We assume that

$$
\varepsilon < \frac{1}{\max_{i \in \{R,L\}, x \in X} |x_i^* - x|}. \tag{40}
$$

This condition implies that, the minimum (with respect to $x \in X$) of the payoffs from entering at some x and winning exceeds the payoff from not entering and losing. The denominator of the right-hand side of (40) is strictly positive because X is a full-dimensional subset of $\mathbb{R}^n$, and it is finite because X is compact.

Suppose that the voters believe that the candidates will implement their ideal policies once they get elected without specifying a policy. That is, we assume $S_i(X, x_j) = S_i(x_i^*, x_j)$, $S_i(x_i, X) = S_i(x_i, x_j^*)$, $S_i(X, X) = S_i(x_i^*, x_j^*)$, and the probability of winning P_i is accordingly defined when X is chosen by at least one candidate. They vote for the candidate whose policy implementation gives them the higher expected payoff. The candidate who attracts more votes will win the election. Given this, we assume that $P_R(x_R^*, x_L^*) = 1$, that is, R will win if neither candidate specifies their policies.[15] The payoff function v_i for each $i = L, R$ is specified accordingly. As in the policy-motivated election campaign in Section G, we assume that the tie is broken in favor of the last candidate to specify the policy if the candidates enter at different times.[16]

Call this game a *persuasion-cost election campaign*. It is characterized by a tuple $(X, \mu, \varepsilon, T, \lambda_L, \lambda_R)$.

Let X^* be the set of policies with which L attracts weakly more votes than R if R does not

---

[15] The case in which $P_R(x_L^*, x_R^*) = 0$ can be analyzed in a symmetric manner, so its analysis is omitted.

[16] We assume such a tie-breaking rule because $(X, \mu) \notin \mathcal{M}$ and thus there is no best response once the opponent enters. As in footnote 80 of the main text, the assumption corresponds to taking a limit of unique PBEs in the models with discrete policy spaces.

specify a policy:

$$X^* = \left\{ \hat{x} : \int_x 1_{\{|x-x_R^*| \geq |x-\hat{x}|\}} \mu(x)\, dx \geq \frac{1}{2} \right\}.$$

In addition, given $x \in X$, let $X^*(x)$ be the set of policies such that R attracts weakly more votes than L given that L enters at x:

$$X^*(x) = \left\{ \hat{x} : \int_{\tilde{x}} 1_{\{|\tilde{x}-x| \leq |\tilde{x}-\hat{x}|\}} \mu(\tilde{x})\, d\tilde{x} \geq \frac{1}{2} \right\}.$$

Given X^* and $X^*(x)$, we can characterize PBE:

Proposition 13. *The persuasion-cost election campaign with $(X, \mu, \varepsilon, T, \lambda_L, \lambda_R)$ has a PBE. Moreover, there exists $t_L^* < \infty$ such that for any PBE, the following hold:*

1. *L enters at $x \in \arg\min_{x \in X^*} |x_L^* - x|$ for $-t > -t_L^*$, while he does not enter for $-t < -t_L^*$.*

2. *R never enters unless L enters. Once L enters at x, R enters as soon as possible at $x' \in \arg\min_{x' \in X^*(x)} |x_R^* - x'|$.*

Candidate R does not have an incentive to enter before L enters since (i) R can win without entering if L cannot obtain an opportunity and (ii) R will lose by entering if L can obtain an opportunity afterward. Given this strategy of R, since L cannot win without entering, he enters if the deadline is near. If the deadline is far, then the probability that R can enter afterward is very large. Hence, entering gives L the payoff close to 0 (or negative if he pays the persuasion cost). Therefore, L does not enter when the deadline is far.

We note that the result that the candidates choose the ambiguous policy can be derived as a corollary of the long ambiguity theorem (Theorem 4). However, we are in "Case 3" of that result, and thus the theorem does not perfectly pin down the equilibrium dynamics for times not too far away from the deadline. Proposition 13 further pins down the equilibrium dynamics for those times as well.

Once we specify x_R^*, x_L^*, and μ, it is straightforward to derive the distribution of the announced policies at the deadline. Thus, we can conduct comparative statics about observable variables.[17]

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[17]The policy to which each candidate enters is generically unique in Proposition 13.

O.2.1 Proof of Proposition 13

Consider a PBE. Given (40), there exists $\bar{t} > 0$ such that for all time $-t \in (\bar{t}, 0]$, L enters at some policy with which he can win. In addition, for each $-t$, if R has already entered, L takes a static best response.

Since R can win without incurring the persuasion cost if L does not enter, we can show that, for each $-t$, R does not enter:

Lemma 21. *Fix candidate L's strategy in which he takes a static best response after R enters. Then, conditional on any history at time $-t$ at which no candidate has entered and R receives an opportunity, not entering is R's unique best response.*

Proof. Since R's not entering until L enters is one of the feasible continuation strategies, it suffices to show that this strategy, denoted by $\bar{\sigma}^R$, is strictly better for R than her entering at $-t$ for each $t \geq 0$.

Fix time $-t$ and a history at time $-t$ such that no candidate has entered. Consider the following two cases:

1. L obtains an opportunity in the time interval $(-t, 0]$. Fix time $\bar{t} > 0$ such that L enters for each $[-\bar{t}, 0]$ if no candidate enters. Conditional on this event, let p be the probability that L obtains an opportunity at some $-\tilde{t} \in [-\bar{t}, 0)$, and then R has an opportunity in some $-\hat{t} \in (-\tilde{t}, 0]$.

   Conditional on this event, entering at x gives R a payoff of $-\varepsilon \, |x - x_R| \leq 0$ while $\bar{\sigma}^R$ gives R a payoff no less than $p \, (1 - \varepsilon \times \max_{x \in X} |x_R^* - x|) > 0$ (strict inequality follows from (40)) since (i) if L has an opportunity at $-\tilde{t} \in [-\bar{t}, 0)$, then either L will have entered by $-\tilde{t}$ or he enters at $-\tilde{t}$, and (ii) if R has an opportunity at some $-\hat{t} \in (-\tilde{t}, 0]$, then she wins for sure by $\bar{\sigma}^R$.

2. L does not obtain an opportunity in the time interval $(-t, 0]$. Conditional on this event, $\bar{\sigma}^R$ gives R a payoff of 1, which is her largest feasible payoff.

Since $\bar{\sigma}^R$ is optimal conditional on each of these two events and the incentive is strict in the first case, it is uniquely optimal for R not to enter given the conditions in the statement of the lemma. □

After L's entry, candidate R enters at the policy x' with which R can win with the lowest persuasion cost:

$$x' \in \arg \min_{x' \in X^*(x)} |x_R - x'|.$$

Given this reaction of R, L's payoff of entering at x at time $-t$ is $e^{-\lambda_R t} - \varepsilon |x_L - x|$. Hence, if he enters, then he enters at the policy with which L can win with the lowest persuasion cost assuming that R will not enter. His payoff of entering at $-t$ is, therefore,

$$e^{-\lambda_R t} - \min_{x \in X^*} \varepsilon |x_L - x|.$$

In contrast, his payoff of not entering at $-t$, given that he will enter as soon as possible in the interval $(-t, 0]$, is

$$\int_0^t \lambda_L e^{-\lambda_L \tau} \left(e^{-\lambda_R(t-\tau)} - \min_{x \in X^*} \varepsilon |x_L - x| \right) d\tau$$
$$= \frac{e^{-\lambda_L t} - \lambda_L e^{-\lambda_R t}}{\lambda_L - \lambda_R} - (1 - e^{-\lambda_L t}) \min_{x \in X^*} \varepsilon |x_L - x|.$$

Let

$$t_L^* = \frac{\log \frac{\lambda_L - (\lambda_L - \lambda_R) \min_{x \in X^*} \varepsilon |x_L - x|}{\lambda_R}}{\lambda_L - \lambda_R} \in (0, \infty)$$

be the smallest t such that L is indifferent between entering and not entering. By the continuity of probabilities in time and boundedness of payoffs, the continuous-time backward induction implies that for $(-t_L^*, 0]$, L enters at $x \in \arg\min_{x \in X^*} \varepsilon |x_L - x|$. It remains for us to show that L does not enter at any time $-t < -t_L^*$. Let σ_L^* be L's strategy such that, at any time $-t$, if R has not entered before $-t$, (i) L does not enter if $t > t_L^*$ and (ii) he enters at some $x \in \arg\min_{x \in X^*} \varepsilon |x_L - x|$ if $t < t_L^*$.

Consider the following two cases:

1. R obtains an opportunity in the time interval $(-t, -t_L^*)$. Conditional on this event, if L enters at x at time $-t$, then L's payoff is $-\varepsilon |x_L - x|$, while σ_L^* gives him a payoff of $e^{-\lambda_R t_L} - \min_{x \in X^*} \varepsilon |x_L - x|$ since no candidate will enter before $-t_L^*$ and L is indifferent between entering and not entering at $-t_L^*$. Hence, it is uniquely optimal not to enter at $-t$.

2. R does not obtain an opportunity in the time interval $(-t, -t_L^*)$. Conditional on this event,

L is indifferent between entering and not entering since he is indifferent between entering and not entering at $-t_L^*$.

Hence, it is uniquely optimal not to enter at $-t$ with $t > t_L^*$.

Overall, we have identified the equilibrium dynamics described in the statement of the proposition.